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(Research Article)

Optimal time trajectory planning and control of space cable robot based on " Brachistochrone " theory

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Abstract

This paper presents optimal-time trajectory planning of Samen cable robot using optimal control theory. The optimal-time trajectory planning solution method is based on the calculus of variations method. This issue can be solved by the Pontryagin's Minimum Principle using the Bang-Bang control. To do so, a time-cost function is considered. By turning optimal-time trajectory planning into a problem-solving procedure with a two-point boundary value problem, the solving stages begin. In addition, the beginning and the end of the path are certain, which can be solved with numerical algorithms. One of the most important and effective methods of minimum time trajectory planning is the brachistochrone method. In this study, the brachistochrone curve is introduced and solved by both brachistochrone method and minimum time trajectory planning using optimal control. First, the path generation for the robot is described using optimization methods; subsequently, with respect to appropriate cost functions, a proportional-integral-derivative controller (PID) and a sliding mode for tracking generated paths are presented. By testing the algorithms in the simulation environment, good and acceptable results are obtained. Simulation results demonstrate proper performance of the proposed method.

Keywords: Optimal control; PID; Sliding mode control; Brachistochrone

1. Introduction

Compared to serial robots, parallel robots, characterized by a unique structure, are more capable to operate. In the meantime, cable parallel robots can be considered as a subset of parallel robots with specific advantages over serial and parallel robots which include a large work space, low moving weight and inertia, high-capacity transfer, high speed and acceleration, low building and maintenance costs, ease of handling and touch applications [1-4].

In cable robots, cable is used as an actuator. Each cable turns around a rotating cylinder and connects to the motor. Cable robots are controlled by the length of the cable. Parallel cable robots are divided into two categories: (1) fully-constrained and (2) under-constrained or suspended. Fully-constrained cable robots have at least one cable more than the number of degrees of freedom, while, for suspended robots, the number of cables is equal to or less than the degrees of freedom of the robot.

In some cases, two points of the workspace are required to remain connected to each other. Optimal time and energy are two important factors in industry. Lately, trajectory planning of cable robots has been studied. Behzadipour and Khajepour [5] studied the path design of a high-speed cable robot. They reduced dimensions of the state space of the system using coordinates of path s, $\dot{s}$. Babrow [6] first proposed this method to design an optimal path for series robots and prescribed paths. Although the use of this technique in trajectory planning of the suspended cable robots makes it

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easy to analyze the analytical relations, in practice, many numerical calculations are required to solve the final optimization problem of the path design. Korayem et al. [7] turned the problem of maximum dynamic load-carrying capacity into a path optimization problem by the Pontryagins Minimization Principle. In this method, any weak initial guess leads to the divergence of responses. Further research has been done [8,9] to design a prescribed path. Trevissani [10] carried out the trajectory planning by considering the speed and acceleration and cable forces' constraints. However, this method is relatively complicated for higher degrees of freedom. The issue of controlling brachistochrone is described in [11]. This solving method is very efficient; however, it has failed to offer a solution to the cable robot problem so far.

In this paper, an efficient method is presented to overcome these shortcomings and is used for optimal-time trajectory planning in a cable robot. The state constraints are determined by considering upper and lower bounds. Through the Pontryagin's principle, the optimal path is obtained. The solving method for two boundary values for the Samen robot cable is given. This article has managed to establish a relationship between optimal control methods and the minimum time solution of the brachistochrone. Considering an increase in degrees of freedom, the design of the path cannot be complicated anymore, making it possible to design a path for the robot.

In the second part of the paper, following the introduction of dynamic equations of the cable robot, two methods of optimal control trajectory planning and brachistochrone curve are considered. In the third section, two theorems are discussed and the relationship between these two methods is determined. In the remainder of the third section, obtained results are proved in the form of two theorems. The fourth part involves providing trajectory planning and simulation methods with MATLAB software. Finally, in the last part of the paper, the results of the simulated responses are given.

2. Minimum time trajectory planning

The main issue discussed in this article is to find a path that will take the robot from the first point to the endpoint in the shortest possible amount of time and with the least control effort. There are also some constraints that should be considered. The block diagram of Figure 1 shows the relationship of the conduction loop with the path curve generator.

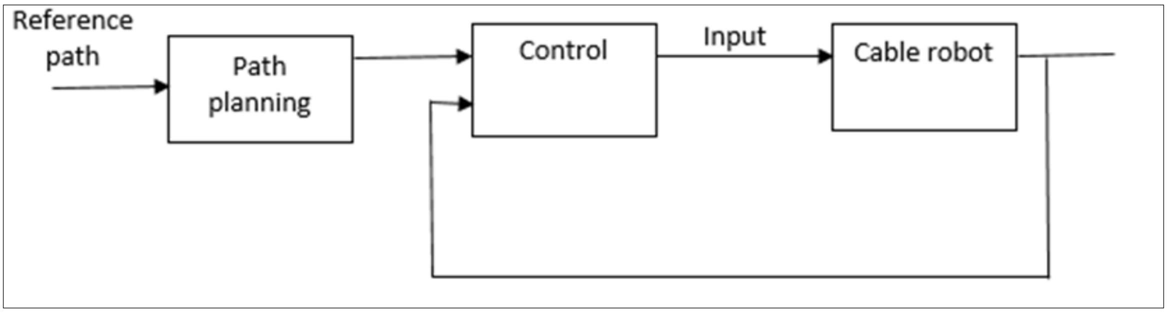

Figure 1 The block diagram of path generation and control

Guiding loop includes a position controller. The position control loop, by receiving a feedback, publishes the input command so that the referenced curve's error vector goes to zero. Therefore, trajectory planning is conducted separately. In this section, the cable robot optimal control techniques are performed to satisfy the dynamic constraints. First, robot dynamical equations are summarized. Then, the trajectory planning begins.

In general, the cable robot model with m cable is obtained from the Newton Euler equations.

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)+\tau_d=-J^T T_C \tag{2.1}$$

$M(q)$ is the inertia matrix of the system, $C(q,\dot{q})$ represents the vector of the Coriolis and centrifuges of the system, and $G(q)$ and τ_d represent the vector of gravity and that of external perturbation (such as wind, etc.), respectively. T_C and J are the vector of cable tensions and the Jacobin matrix of the system, respectively.

For the Samen cable robot, which has three degrees of transitional freedom, the matrices and vectors are presented as follows:

$$M(q) = diag(m, m, m),\ \ C(q, \dot{q}) = 0\ ,\ \ G(q) = [0, 0, -mg]^T \quad (2.2)$$

M is the mass of end effector, or the camera, and g is the acceleration of the gravity of the earth. According to the explanation given, the goal is finding an analytical optimal solution for the cable robot to the specified point so that $J = \int dt$ function and the path curve can be minimized and calculated, respectively. In fact, the problem is solved for the initial and final conditions. Among the methods presented to solve the optimal time problem, two cases are proposed.

2.1 Brachistochrone Method

The Brachistochrone problem concerns finding a curve to cross a point to another under gravity in the shortest amount of time. Consider two points A and B, as shown in Figure 2, where point A is not lower than B point. If the object with zero initial velocity is released on the curve below at point A, the object moves through the gravitational force toward B. The goal is to calculate the trajectory to optimize the time of movement. This problem can be solved by calculus of variations method [11].

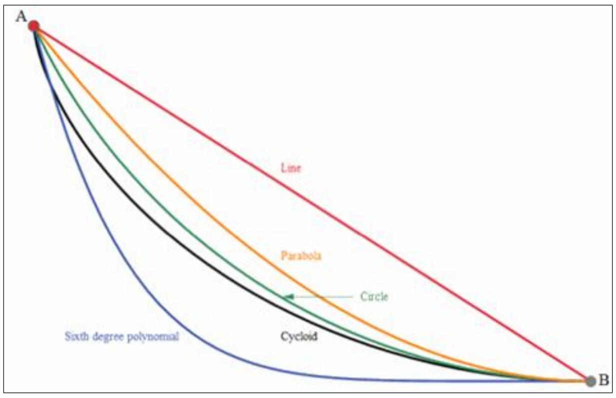

Figure 2 Brachistochrone curve

The purpose is to determine path z (x) to minimize the arrival time from points A to B. In (2.3), ds is the path element and v is the speed of the robot's motion. For this purpose, the arrival time is from relations (2.3) to (2.6) is calculated.

$$ds = vdt$$

$$v = \sqrt{2g(z - z_A}$$

(2.3)

(2.4)

$$ds = \sqrt{dx^2 + dz^2 = \sqrt{1 + z^2 dx}}$$

(2.5)

$$t_{AB} = 1 / \sqrt{2g} \int_A^B \frac{\sqrt{1+z'^2}}{\sqrt{z-z_A}} dx \quad (2.6)$$

where g is the acceleration of gravity, (z_A, x_A) represent the initial coordinates of the object on the vertex plane, and (z_B, x_B) are the coordinates of the end of the object on the vertex plane.

Minimizing the integral in the above equations means minimizing the time function. Therefore, the cost function is defined as in relation (2.7):

$$J = \int_{B}^{A} f\left(z, z'\right) dx \tag{2.7}$$

By placing (2.6) into (2.7), the cost function is in the form of (2.8):

$$f\left(z, z'\right) = \frac{\sqrt{1+z'^2}}{\sqrt{z-z_A}} \tag{2.8}$$

Since function f (z, z) is independent of x, the Euler equations can be used as in relations (2.9) to (2.11):

$$f_z = \frac{-1}{2}\sqrt{\frac{1+z'^2}{\left(z-z_A\right)^3}} \tag{2.9}$$

$$f_{z'} = \frac{z'}{\sqrt{\left(z-z_A\right)\left(1+z'^2\right)}} \tag{2.10}$$

$$f - zf_{z'} = \frac{1}{\sqrt{\left(z-z_A\right)\left(1+z'^2\right)}} = \frac{1}{\sqrt{2r}} \tag{2.11}$$

Where c value is replaced by 1 / √2r. The new variable θ is defined as in relation (2.12):

$$z' = cot\frac{\theta}{2} = \frac{\sin\theta}{1-\cos\theta} \tag{2.12}$$

By replacing θ into equation (2.11), we obtain:

$$z - z_A = \frac{2r}{1+z'^2} = 2r\cos\frac{\theta}{2} = r(1-\cos\theta) \tag{2.13}$$

Through derivation of (2.13), relation (2.14) is obtained in relation to θ:

$$\frac{dz}{d\theta} = r\sin\theta \tag{2.14}$$

Using the derivative of the chain rule, relation (2.15) is obtained as follows:

$$\frac{dx}{d\theta} = \frac{dx}{dz}\frac{dz}{d\theta} = \frac{1-\cos\theta}{\sin\theta} r\sin\theta = r(1-\cos\theta) \tag{2.15}$$

By integrating (2.15), position x is calculated as follows:

$$x=\int r(1-\cos\theta)d\theta = r(\theta-\sin\theta)+a \tag{2.16}$$

where a is constant in integration (2.16). This trajectory is a part of a cycloid. The equations of this curve are rewritten in (2.17).

$$\begin{aligned} x-x_A &= r(\theta-\sin\theta) \\ z-z_A &= r(1-\cos\theta) \end{aligned} \tag{2.17}$$

Solving this problem is limited to a curve called the cycloid curve, where two parameters θ and r have the following range:

$$\begin{aligned} &0 \le \theta \le 2\pi \\ &0 \prec r \end{aligned} \tag{2.18}$$

where θ is a real parameter that represents the radius angle of the circular circle measured in radians, and r is the radius of the circle. The cycloid equation can be expressed in terms of (2.19):

$$x = r\cos^{-1}(1-\frac{y}{r}) - \sqrt{y(2r-y)} \tag{2.19}$$

After reviewing the method of brachistochrone, the optimal control methods and the Hamiltonian equation formation are considered in terms of the cost function in order to obtain the optimal trajectory.

2.2 Trajectory planning with optimal control

In the optimal control problem, the objective is to determine the state function and control so as to optimize the defined function.

The objective function for this system is defined as in the relationship (2.20) [12].

$$J = \int_{t_0}^{t_f} g \, dt = \int_{t_0}^{t_f} \left(1+\varepsilon^2\left(\tau_1^2+\tau_2^2+\tau_3^2+\tau_4^2\right)\right)dt \tag{2.20}$$

where τ_i is the torque of the motors. In this bi-functional objective function, both time and control signal are optimized, simultaneously. Ɛ is a weighting factor and, when is set to zero, the objective function only includes time optimization.

The problem is to find $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix}$ where function $J = \int (1+\varepsilon^2\tau^2)dt$ that is bound to the equations of state and boundary conditions; besides, the following constraints are minimized:

$$x(t_f) = (5.2, 5.8, -0.55)$$

$$x(0) = (5, 6, -0.05)$$

$$v(0) = (0, 0, 0)$$

$$\mathrm{v}(\mathrm{t}_f) = (0,0,0)$$
$$\tau_{i\min} \le \tau_1, \tau_2, \tau_3, \tau_4 \le \tau_{i\max}$$
$$v_{\min} \le v(t) \le v_{\max}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ \frac{1}{m}F_1 \\ \frac{1}{m}F_2 \\ \frac{1}{m}(F_3 + mg) \end{bmatrix} \tag{2.21}$$

The Hamiltonian equation is shown in the following:

$$g = 1 + \varepsilon^2 \tau^2$$

$$H = g + \lambda_1 x_4 + \lambda_2 x_5 + \lambda_3 x_6 + \lambda_4 \frac{1}{m} F_1 + \lambda_5 \frac{1}{m} F_2 + \lambda_6 \frac{1}{m}(F_3 + mg) \tag{2.22}$$

According to the Pontryagin's principle concerning the optimal path, the optimal conditions obtained by deriving the Hamiltonian function with respect to state, pseudo-state, and control data should be satisfied.

Co-state equations of relationships are given by:

$$\dot{\lambda}_i + \frac{\partial H}{\partial x_i} = 0 \tag{2.23}$$

However, the other condition for optimality is to have the following relationship:

$$H(x^*, u^*, \lambda^*, t) \ge H(x^*, u, \lambda^*, t) \tag{2.24}$$

It is essential that this inequality be set to: $\frac{\partial H}{\partial \tau} = 0$. By using this equation, torques are calculated.

For the three-cable robot, twelve differential equations are obtained; to solve them, twelve boundary conditions are required (six conditions for time t_0 and six conditions for time t_f). Solving the above equations is performed by the bvp4c command of the MATLAB software; besides, the optimal values of the torques are obtained. In order to illustrate the relationship between the two methods, two very important results are obtained, which are presented in the form of two theorems.

Theorem 1: The solution obtained from the minimum time trajectory planning method present in part B is optimized.

Proof: Suppose that u^* is the optimal control for the minimum time problem which is bound to equations (2.21) calculated from the condition (2.25).

$$\frac{\partial H}{\partial \tau} = 0 \tag{2.25}$$

As a result, optimum u is obtained in terms of state and quasi-state variables.

In addition, the quasi-states or Lagrange coefficients are obtained with respect to this control signal. By placing this control signal into the equations of state space of the robot, the equations are complete. Ultimately, the Hamiltonian equation is solved and achieved:

$$H = 1 + \varepsilon^2\tau^2 + \lambda_1 x_4 + \lambda_2 x_5 + \lambda_3 x_6 + \lambda_4 \frac{1}{m} F_1 + \lambda_5 \frac{1}{m} F_2 + \lambda_6 \frac{1}{m}(F_3 + mg)$$

Through the instrumentality of the Hamiltonian principle and based on the Pontryagin theory, u is obtained such that the objective function can be minimized.

To solve (2.23) , 2n differential equation will have (2n=12) for state equations and quasi-state with boundary conditions that are solved by converting the mentioned problem into a two-point boundary value problem.

As a result, the obtained answer is optimal according to Pontryagin's law.

Theorem 2: The path obtained from the optimal-time trajectory planning method is a Brachistochrone curve.

Proof: According to Theorem 3.1, the optimal-time path curve is obtained. By solving differential equations for states, the robot's position is obtained at any moment. Using the approximation of LS, the equation of the path is approximated according to the obtained positions. With respect to the obtained equations, the equations are attained in the brachistochrone curve as follows In other words, the path equations are in the form of equation (2.26):

$$\begin{aligned} x - x_a &= r(\theta - \sin\theta) + \mathrm{a} \\ y - y_a &= r(1 - \cos\theta) \end{aligned} \tag{2.26}$$

In the case of our problem with boundary conditions, coefficient r is equal to 0.82. The path equation for this problem is as follows:

$$\begin{aligned} x &= 0.82(\theta - \sin\theta) - 0.15 \\ y &= 0.82(1 - \cos\theta) - 0.05 \end{aligned}$$

$$x = 0.82\cos^{-1}(1 - \frac{y}{0.82}) - \sqrt{y(1.64 - y)}$$

■

The proposed optimal solution results in the same brachistochrone curve. The results obtained in the optimal method for the two boundary values are given in this paper; in other words, the optimal path curve is the same as the brachistochrone curve. With the approximation of this curve in the form of the Brachistochrone path equations, the optimum time curve obtained with a good approximation and the lowest possible error to the curve is presented in the brachistochrone method.

In the Brachistochrone method, some hypotheses can be applied herein, too. For example, there is no friction in both methods. The force is only the gravitational force. Both methods minimize the time. In addition, in both of these methods, the beginning and end points are specified and the final time is uncertain.

3. Simulation

In Figure 3, there are three curves: optimal, brachiochrone, and linear. As is clear, two optimal curves and brachiochrone are very close together.

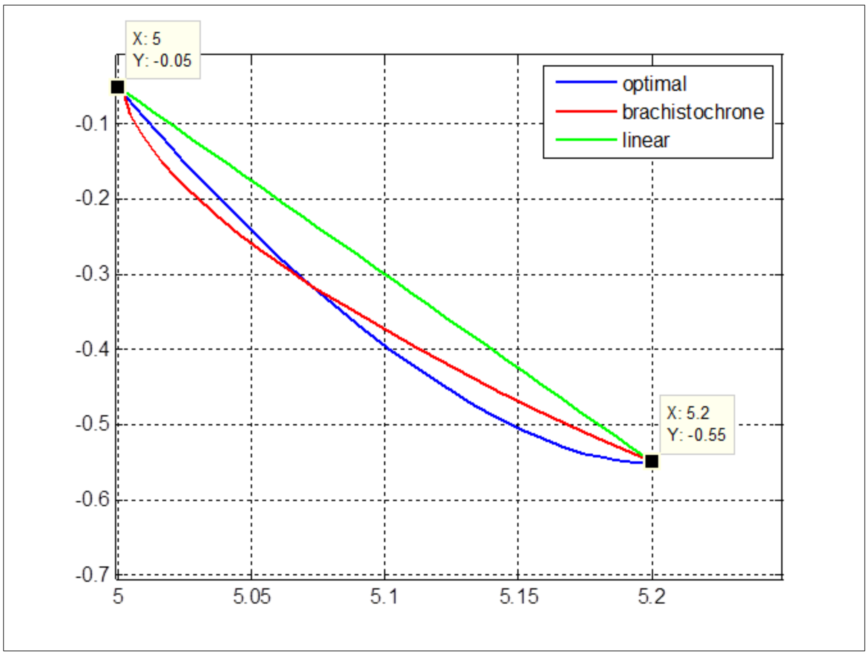

Figure 3 Comparison of two optimal and brachiochrone curves with linear curves

The difference between two optimal curves and brachiochrone is also plotted in Figure 4. According to the simulation results, it would not be unexpected to find that the two path curves are the same with the good approximation.

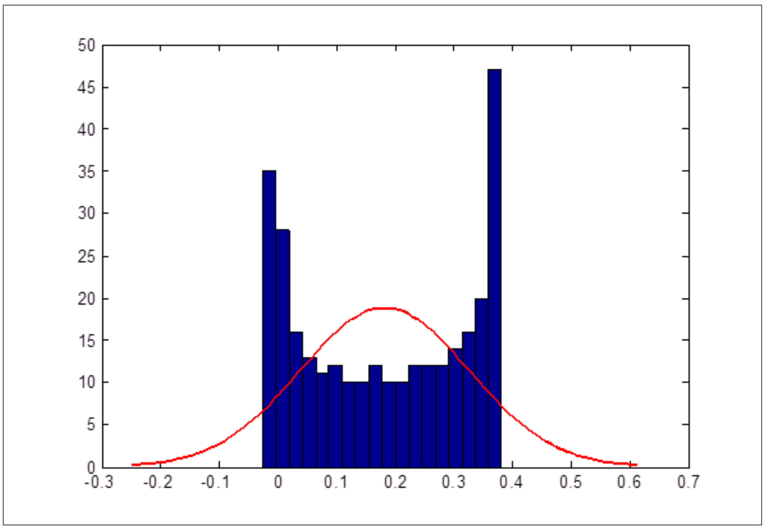

Figure 4 Approximation error of two curves

Table 1 shows the time required to reach the destination for three different linear modes, brachistochrone, and an optimized method.

The required time in the optimal control method is obtained using the fmincon command in MATLAB with respect to the cost function and path constraints. In the brachistochrone method, this time is calculated using formula (2.6). According to the Table, the optimal curve obtained from this paper reaches the final state almost at the same time from the initial state.

Table 1 Comparison of the times required to reach the goal

Control method	linear	brachistochrone	Optimal control
The required time	2.2	1.78	1.8042

According to the above figures, the optimal path obtained is the same as the brachistochrone curve obtained in the previous sections.

Two PID controllers and a sliding mode for this path are presented herein. In this section, the comparison between the two controllers is investigated.

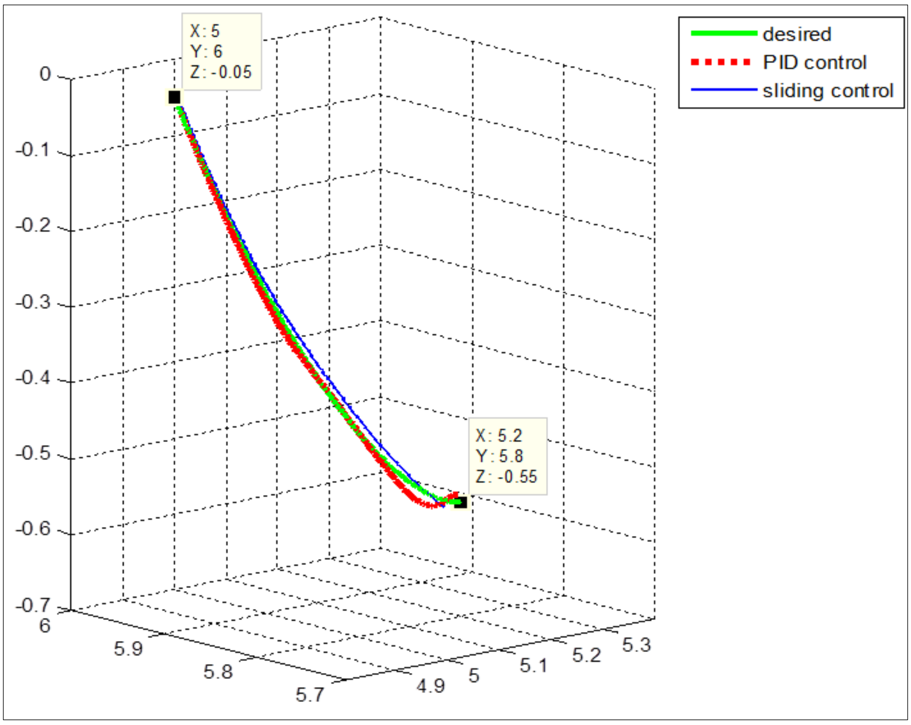

Figure 5 Comparison of two sliding modes and PID controllers

According to Figure 5, both tracking controllers have performed quite well. Figure 6 shows the tracking error of the two controllers.

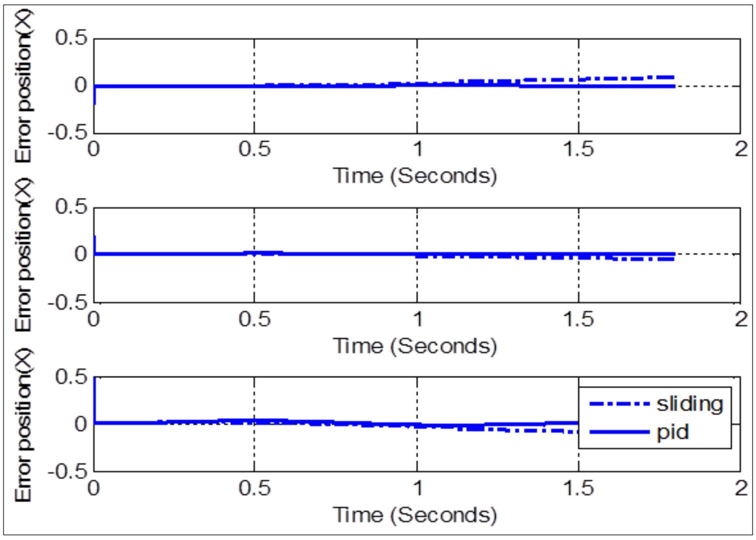

Figure 6 Comparison of two modes of sliding mode controller and PID

Now, two criteria are defined for error in order to illustrate the subject.

$$e = y - y_{desired}$$

$$SSE(e) = \sum_1^n e_i^2$$

$$MAE(e) = \frac{\sum_1^n |e_i|}{n} \qquad (3.1)$$

where e is an error. SSE and MAE represent the sum of squared errors and the mean absolute error value, respectively.

By calculating mean absolute error (MAE) for both controllers, the obtained numbers for two sliding modes and PIDs are 0.0359 and 0.0393, respectively. Therefore, as a result of the sliding mode controller, the function is more efficient than the PID controller in the absolute value field. Indeed, by calculating the level below the graph in the error diagram, the sums of the squared errors (SSE) for the sliding mode and PID controllers are 2.1324 and 2.0285, respectively. In other words, thanks to this criterion, the PID controller performs better. This difference can be justified based on Figure 5. Because, at the end of the path, the PID controller shows a better performance in tracking the path. Table 2 displays the comparison of the errors of these two controllers.

Table 2 Comparison of errors of the two controllers

criterion	SSE	MAE
PID	2.4547	0.0393
SMC	2.0285	0.0359

4. Conclusion

The trajectory planning problem for the Samen cable robot was introduced using optimal control methods. In the current paper, the time of robot's movement from the beginning to the end was considered as a cost function to minimize it. In addition, in the trajectory planning, constraints of the path were considered as torque. The optimal path and equations were solved using the bang-bang control with two nonlinear boundary values. In another section of the Brachistochrone curve, the answer was minimum time, and the comparison and combination of these two methods led to the proving of two theorems. The simulation section describes the optimal path for the Samen cable robot in the shortest possible amount of time.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest.

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