

(RESEARCH ARTICLE)



The application of modern meta-heuristic algorithms for solving complex optimization problems

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International Journal of Scholarly Research in Engineering and Technology, 2023, 02(01), 065–075

Publication history: Received on 09 February 2023; revised on 22 March 2023; accepted on 24 March 2023

Article DOI: <https://doi.org/10.56781/ijret.2023.2.1.0021>

Abstract

This paper applies two new meta-heuristic algorithms proposed in 2022 for solving different optimization problems, including the driving training-based algorithm (DTBA) and the average and subtract-based algorithm (ASBA). The considered optimization problems employed in this paper are characterized by different quantities of the dimensions and different involved constraints at various degrees of complexity. The results obtained by the two algorithms are illustrated by three types of convergences, including the minimum, average, and maximum convergences. By analyzing the results obtained by the two applied methods on four different optimization algorithms, DTBA proved itself to be the better applied method over ASBA. Particularly, DTBA has reached the optimal fitness value much faster than ASBA, regardless of how complicated the optimization problem is. From these analyses, DTBA is acknowledged to be the effective algorithm for dealing with the considered optimization problems.

Keywords: Optimization problem; Meta-heuristic algorithms; Driving training-based algorithm; Average and subtract based optimization

1 Introduction

Optimization problems are the most common problems, which are easy to recognize in both economics, engineering, and other fields of human life. The determination of an optimal solution for a specific optimization problem will offer a lot of advantages and reduce the use of essential resources. An optimal solution will result in the best fitness value for the considered optimization problem. In accordance with the initial target, the best fitness value can be the maximum or minimum value. By acknowledging the importance of finding the optimal solution to optimization problems, various optimization methods are developed and implemented. These methods can be separated into two main groups, including the classical search methods and the modern search methods. Typical methods of the first group can be named, such as the Lagrange methods [1-3], Newton methods [4-5], quadratic programming [6-7], gradient search method [8-10], etc. These classical methods have several common drawbacks, as follows: 1) slow response; 2) requiring a sequence of complex calculations; 3) being unreliable while solving the complicated constraints featured by the considered problem; and 4) being unfeasible and unapplicable for large-scale optimization problems. To fix all these drawbacks, meta-heuristic algorithms are being developed to be the game changer for dealing with high-degree complex and large-scale optimization problems. Because of these advance features, a lot of meta-heuristics are applied to determine the optimal solution to a wide range of optimization problems, such as particle swarm optimization (PSO) [11], evolutionary programming (EP) [12], cuckoo search algorithm (CSA) [13], harmony search algorithm (HSA) [14], lion optimization algorithm (LOA) [15], coyote optimization algorithm (COA) [16], Archimedes optimization algorithm (AOA) [17], ant colony optimization (ACO) [18], bat algorithm (BA) [19], genetic algorithm (GA) [20], chaotic game optimization (CGO) [21], and crystal structure algorithm (CRSA) [22]. By fully understanding the advantages of the meta-heuristic algorithm, two novel meta-heuristic algorithms, including the driving-trained based algorithm (DTBA)

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[23] and the average and subtract-based algorithm (ASBA), will be simultaneously applied to find the optimal solution to different optimization problems.

Speaking of optimization problems, they are normally structured by an objective function and the constraints. More specifically, the objective function is constituted by different variables. These variables consist of two types: the control variables and the dependent variables. The control variables are generated at the beginning of the optimal process by the optimization methods, while the dependent variables are determined after all the control variables are legally created. Next, in terms of the constraints, there are also two types, including the inequal constraints and the equal constraints. Normally, the inequal constraints are used to limit the ranges of the control variable. That means that all the control variables are legally generated if they exist within their limits. On the contrary, equal constraints are mostly used to determine independent variables. Note that, in the optimal process, the dependent variables can sometimes violate their limits. Consequently, the solution that contains the violated dependent variables cannot be accepted as a valid solution. On the contrary, the solution with both control variables and legally dependent variables is considered to be the feasible solution for the given optimization problem. Lastly, a fitness function must be established before the implementation of the meta-heuristic algorithms takes place to solve the given problem. A fitness function generally consists of an objective function, as mentioned earlier, and a penalty term. A penalty term is used to point out how much the dependent variable violates its limit.

This paper focuses on solving the complex and nonlinear optimization problems. These problems are constituted by different objective function accompany with various constraints. The specific mathematic models of the optimization problems studied in this paper will be described in the next section. The main contribution of this paper is as follows:

- Apply two novel meta-heuristic algorithms, including the Driving train – based algorithm (DTBA) and the Average and Subtract – based algorithms (ASBA) to different complex optimization problem.
- Determine the best algorithm for solving the given problems between the two applied algorithms.
- Demonstrate the superiority of the new algorithms while compared to the previous ones

In addition to the introduction in section 1, section 2 will describe the optimization problems and the involved constraints, section 3 briefly introduces about the two applied algorithms, section 4 presents the results obtained by applying the two algorithms for the given problem in section 2, and finally section 5 reveals the main conclusions

2 Problem formulation

2.1 The general description of the optimization problem

In this section, we use different theoretical optimization problems to test the raw performance of the applied algorithms. In general, all the selected optimization problems are formulated in the common expression as follows:

$$F(a_1, a_2, a_3, \dots, a_n) \dots\dots\dots (1)$$

Where, F is the objective function featured by the optimization problem; $a_1, a_2, a_3, \dots, a_n$ are the involving variables, with n is the number of the dimensions.

Besides, these optimization problems are always accompanied by the two main constraints, including the inequal constraints and equal constraint. The expressions of these typical constraints are given as follows:

2.1.1 The inequal constraints

As mentioned earlier, the inequal constraints are mainly used to define the boundaries of the control variables. Suppose that a_2, a_3, \dots, a_n are selected to be the control variables, the expressions of the inequal constraints for these variables will be given as the following equations:

$$a_2^{min} \leq a_2 \leq a_2^{max} \dots \dots \dots (2)$$

$$a_3^{min} \leq a_3 \leq a_3^{max} \dots \dots \dots (3)$$

$$a_n^{min} \leq a_n \leq a_n^{max} \dots \dots \dots (4)$$

Where, a_2^{min} , a_3^{min} , and a_n^{min} are the lowest boundaries of the control variables a_2 , a_3 , and a_n ; a_2^{max} , a_3^{max} , and a_n^{max} are the highest boundaries of the selected control variables.

2.1.2 The equal constraints

While all the control variables are fully generated, the equal constraints is applied to determine the dependent variable. Generally, the equal constraints are typically established to describe the relationship between all the variables as given below:

$$Ha_1 - Ka_2 + Ma_3 + \dots - Na_n = T \dots \dots \dots (5)$$

Where, H, K, M, N, T are the given coefficients.

2.2 The particular optimization problems applied in the paper

In this subsection, we will use the set of four different optimization problems to test the real efficiency of the two applied meta-heuristic algorithms. The mathematical expression of each optimization problem and its involved constraint is respectively present as follows:

2.2.1 The first optimization problem

The first optimization problem is described as follows:

$$F_1(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^2 (x_i - a_{ij})} \right)^{-1} \dots \dots \dots (6)$$

With

$$-65.53 \leq x \leq 65.53 \dots \dots \dots (7)$$

$$a = [-32 \ -16 \ 0 \ 16 \ 32 \ -32 \ -16 \ 0 \ 16 \ 32 \ -32 \ -16 \ 0 \ 16 \ 32 \ -32 \ -16 \ 0 \ 16 \ 32 \ -32 \ -16 \ 0 \ 16 \ 32 \ -32 \ -16 \ 0 \ 16 \ 32; -32 \ -32 \ -32 \ -32 \ -32 \ -32 \ -16 \ -16 \ -16 \ -16 \ -16 \ 0 \ 0 \ 0 \ 0 \ 16 \ 16 \ 16 \ 16 \ 16 \ 16 \ 32 \ 32 \ 32 \ 32 \ 32] \dots \dots \dots (8)$$

2.2.2 The second optimization problem

The second optimization problem is expressed by the following expressions below:

$$F_2(x) = \sum_{i=1}^m -x_i \times \sin(\sqrt{|x_i|}) \dots \dots \dots (9)$$

With

$$-500 \leq x \leq 500 \dots \dots \dots (10)$$

2.2.3 The third optimization problem

The mathematical model of the third optimization problem is described as below:

$$F_3(x) = -20 \times \exp\left(-0.2 \sqrt{\frac{1}{m} \sum_{i=1}^m x_i^2}\right) - \exp\left(\frac{1}{m} \sum_{i=1}^m \cos(2\pi x_i)\right) + 20 + e \dots \dots \dots (11)$$

With

$$-32 \leq x \leq 32 \dots \dots \dots (12)$$

2.2.4 The fourth optimization problem

The fourth optimization problem is modeled by the following equation:

$$F_4(x) = \sum_{i=1}^{11} \left[a_i - \frac{x_1 \times (b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4} \right]^2 \dots \dots \dots (13)$$

With

$$-5 \leq x \leq 5 \dots \dots \dots (14)$$

And

$$\begin{aligned}
 a &= [0.1957, \quad 0.1947, \quad 0.1735, \quad 0.16, \quad 0.0844, \quad 0.0627, \quad 0.0456, \\
 &\quad 0.0342, \quad 0.0323, \quad 0.0235, \quad 0.0246]; \\
 b' &= [0.25, \quad 0.5, \quad 1, \quad 2, \quad 4, \quad 6, \quad 8, \quad 1, \quad 0, \quad 1, \quad 2, \quad 1, \quad 4, \quad (15) \\
 &\quad 1, \quad 6]; \\
 b &= \frac{1}{b'}; \dots\dots\dots
 \end{aligned}$$

3 The applied methods

3.1 The Driving training-based optimization

The Driving training-based algorithm (DTBA) is inspired by the driving practice of achieving driving license in human life. The driving practice is divided into different stages, and these stages are the mainstay of the update process for new solutions belong to DTBA. The mathematical expression of these stages will be given as follows:

3.1.1 The first stage

The expression model of the first stage is mathematically formulated as follows:

$$S_n^{new,S1} = \begin{cases} S_n + \delta \times (RS_l - RN \times S_n), & \text{if } F_{RS_l} < F_{S_n} \\ S_n + \delta \times (S_n - RS_l), & \text{else} \dots\dots\dots \end{cases} \quad (16)$$

Where, $S_n^{new,S1}$ is the new solution updated in the first stage, $m = 1, \dots, PN$ and PN is the population number; δ is stochastically generated between 0 and 1; RS_l is the driving trainer for the learner $l = 1, \dots, AT$ and AT is the quantity of the driving trainers; RN is stochastically generated between 0 and 1; F_{RS_l} and F_{S_n} is the fitness value given by RS_l and the current solution S_l .

The quantity driving trainer AT is determined as shown below:

$$AT = 0.1 + PN \times \left(1 - \frac{IT}{IT^{max}}\right) \dots\dots\dots \quad (17)$$

Where, IT and IT^{max} are the present iteration and the highest preset iteration.

3.1.2 The second stage

In the second stage, the new solution are updated by using the expression below:

$$S_n^{new,S2} = CI \times S_n + (1 - CI) \times RS_l \dots\dots\dots \quad (18)$$

Where, $S_n^{new,S2}$ is the new solution update in Stage 2, $n = 1, \dots, PN$ and PN is the population number; CI is the comparative indicator and CI is resulted by using the following equation:

$$CI = 0.01 + 0.9 \times \left(1 - \frac{IT}{IT^{max}}\right) \dots\dots\dots \quad (19)$$

3.1.3 The third stage

All the solutions in the third stage are updated by using the following expression:

$$S_n^{new,S3} = S_n + (1 - \delta) \times NF \times \left(1 - \frac{IT}{IT^{max}}\right) \dots\dots\dots \quad (20)$$

Where, $S_n^{new,S3}$ is the new solution updated in the third stage; NF is the narrowing factor.

3.2 The Average and subtraction-based optimizer

ASBA uses the average and the results by subtracting the best and the worst individual to drive the whole update process reach the optimal solution. ASBA also utilized three stages to complete the update process for new solutions.

3.2.1 The first stage

On the first stage, the new solutions are updated using the following equation:

$$X_a^{new,s1} = \begin{cases} X_a + \theta \times (AV^{S1} - PI * X_a), & \text{if } F(AV^{S1}) < F(X_a) \\ X_a + \theta \times (X_i - AV^{st1}), & \text{else} \end{cases} \dots\dots\dots (21)$$

With

$$AV^{S1} = \frac{X_{HS} + X_{LS}}{2} \dots\dots\dots (22)$$

In the Equations (21) – (22) above, $X_a^{new,s1}$ is the new solution tupdate in the first stage, $a = 1, \dots, PN$ and PN is the population number; θ is randomly generated between 0 and 1; PI is period indicator; AV^{S1} is the average solution of the whole population; A_{hq} and A_{lq} are, respectively, the best and the worst solution.

3.2.2 Stage 2

After that, all solutions of the population will be update following the equation below:

$$X_a^{new,s2} = X_a - \theta_2 DS^{st2} \dots\dots\dots (23)$$

With

$$DS^{st2} = X_{HS} + X_{LS} \dots\dots\dots (24)$$

Where, $X_a^{new,s2}$ is the new solution tupdated inthe second stage, $a = 1, \dots, PN$ and PN is the population number; DS^{st2} is the distinctive solution.

3.2.3 Stage 3

On the third stage, all the solution are updated by executing the following expression:

$$X_a^{new,s3} = X_i - \theta_3 (X_a - PI * X_{HS}) \dots\dots\dots (25)$$

Where, $X_a^{new,s3}$ is the new solution a updated in stage, θ_3 is randomly generated between 0 and 1.

4 The results

In this section, both DTBO and ASBO will be applied to determine the optimal solutions to the four optimization problems as mentioned in subsection 2.2. After that, the results will be discussed, analyzed, and compared with each other to find out which method is the best applied method for the given optimization problems. To conduct a fair comparison, we use the same control parameters for the initial population (PN), maximum number of iterations (IT^{max}), and number of independent runs (Run). Particularly, these control parameters are respectively set at 30, 500, and 50.

All the work in this paper was conducted on a personal computer with the basic specifications, including a 2.6 GHz central processing unit (CPU) and 8 GB of random-access memory (RAM). The complete coding and related simulation are performed using MATLAB programming language version 2018a.

4.1 Results obtained from the first optimization problem

For the first optimization problem, the results obtained by DTBA and ASBA are compared through different criteria, including the minimum convergence, average convergence, and the maximum convergence. In Figure 1, 2, and 3, the blue lines represent the convergence achieved by DTBO, while the red ones represent the similar convergences drawn by ASBA after its execution. By taking a look in figures 1 and 2, DTBA proves the better response capabilities than ASBA by reaching the optimal results much faster. Specifically, in Figure 1, DTBA needs less than 40 iterations for determining the best fitness values of the F1, meanwhile ASBA uses over 65 iterations for reaching the same results. However, ASBA show its better performance over DTBA in Figure 3. Particularly, in the figure, the maximum convergence given by ASBA is located in the lower position than DTBA. To sum up on this case, DTBA have shown its advantage over ASBA, but this advantage is relatively small due to the characteristics of the given optimization problem.

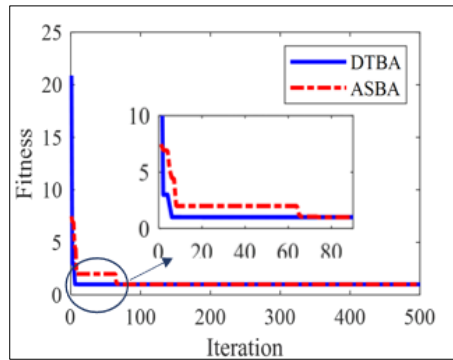


Figure 1 The minimum convergences obtained by DTBA and ASBA

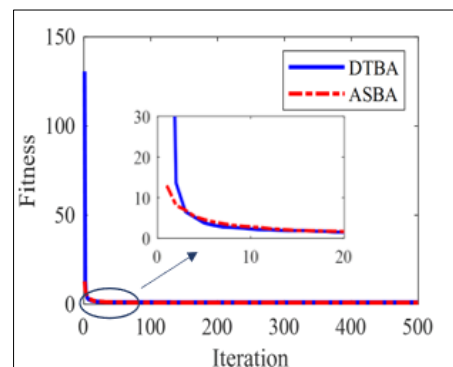


Figure 2 The average convergences obtained by the DTBA and ASBA

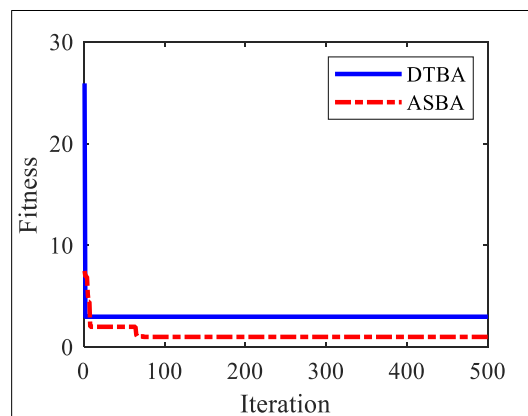


Figure 3 The maximum convergences obtained by DTBA and ASBA

4.2 Results obtained from the second optimization problem

In this section, a trigonometric optimization problem is used to continuously test efficiency of the two applied method. Besides, the boundaries of the variables contained in the optimization problem is substantially enlarged while compared with the first objective function. This enlargement also means that, the scale of the problem is much bigger and therefore, the search for the optimal solution in search space will be more difficult.

This section uses the same legends as the results of section 4.1. In the Figures 4, 5 and 6, DTBA completely outperforms ASBA in all comparisons criteria. Particularly, DTBA always reaches the optimal fitness values extremely faster than ASBA, while DTBA cannot reach any of optimal values in all three type of convergences. In conclusion, the superiority of DTBA over ASBA while solving this optimization problem is very clear and undeniable.

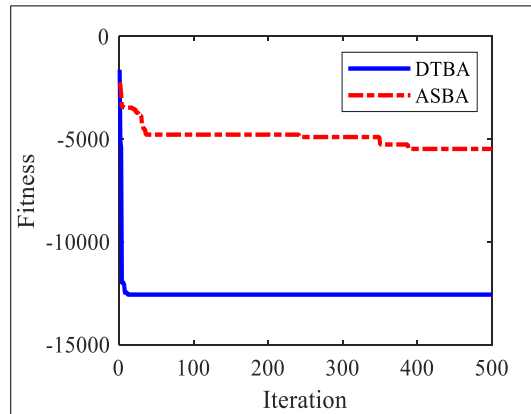


Figure 4 The minimum convergences obtained by DTBA and ASBA

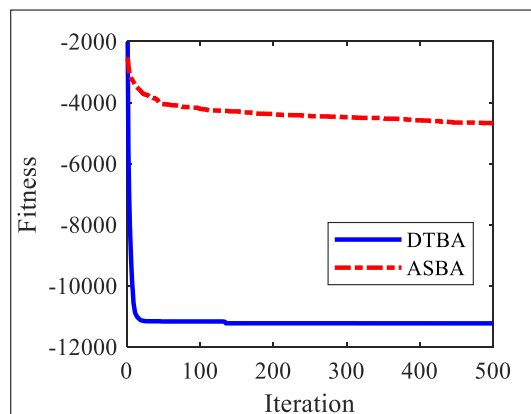


Figure 5 The average convergences obtained by the DTBA and ASBA

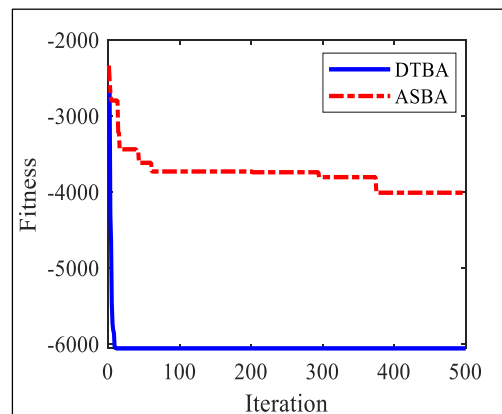


Figure 6 The maximum convergences obtained by DTBA and ASBA

4.3 Results obtained from the third optimization problem

In this section, another trigonometric optimization problem is employed to investigate the efficiency of the two applied methods. Unlike the test problem in Section 4.2, this one does not have a large number of variables, but the optimization is highly complex due to the presence of the exponential term and the square root element. As shown in Figures 7, 8, and 9, the DTBA still maintains its superiority over the ASBA in all criteria. For the minimum convergence, DTBA still reaches the optimal fitness value faster than ASBA, although the difference is not much. Next, in the two remaining convergences, DTBA rapidly rounds up to the optimal value with fewer iterations than ASBA. Clearly, DTBA has shown great capability to solve the highly complex optimization problem. Besides, ASBO also provides a surprising

performance in this test, although its performance is a bit disadvantageous when compared to DTBA. By this evidence, DTAB is no doubt the effective applied method for the given problem.

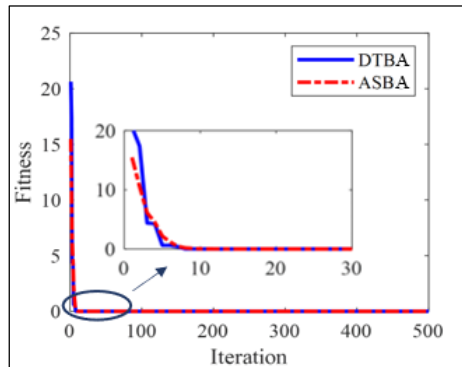


Figure 7 The minimum convergences obtained by DTBA and ASBA

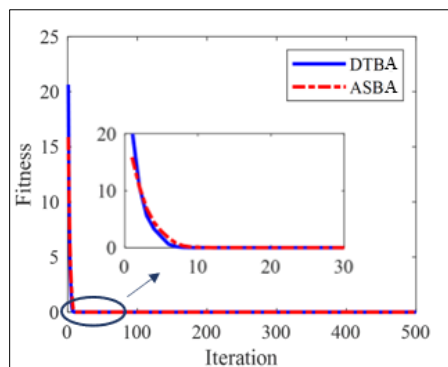


Figure 8 The average convergences obtained by the DTBA and ASBA

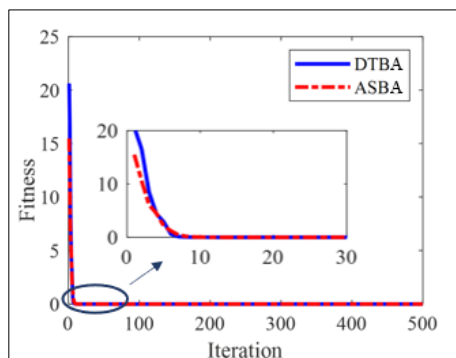


Figure 9 The maximum convergences obtained by DTBA and ASBA

4.4 Results obtained from the fourth optimization problem

In this section, a polynomial optimization problem is utilized to validate the efficiency of both DTBA and ASBA. This problem is described by the sum of a quadratic expression. The optimal solution is a set of four variables whose values are allowed to vary in the interval between -5 and 5. Moreover, there is also the presence of the two given coefficients, a and b, and their values are defined by the matrices a and b, respectively. Similar to the optimization problem in Section 4.3, the optimization problem in this section does not accompany a large-scale search space, but it is highly complex due to the use of many variables along with the varied coefficients.

Regardless of many adversaries, as mentioned above, DTBA continuously provides a surprising performance over ASBA. Specifically, DTBA only requires less than 10 iterations for reaching the optimal value of fitness, while ASBA must utilize more than 30 iterations for achieving the similar one. The same phenomenon can be seen with the average and

maximum convergences, where DTBA can achieve the optimal values with fewer iterations than ASBA. In conclusion, DTBA still provides great superiority over ASBA while solving such a complex problem as given in this section.

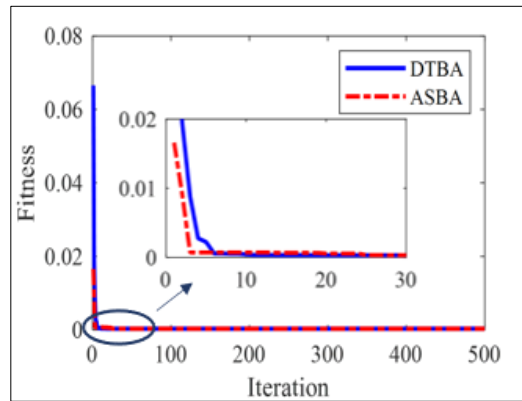


Figure 10 The minimum convergences obtained by DTBA and ASBA

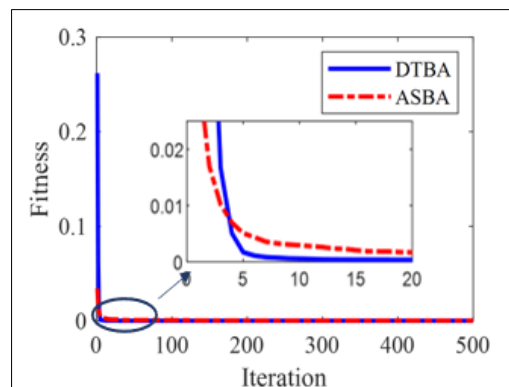


Figure 11 The average convergences obtained by the DTBA and ASBA

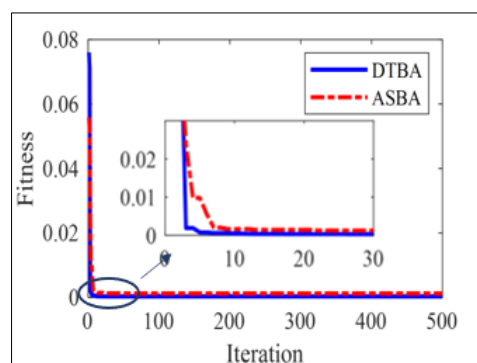


Figure 12 The maximum convergences obtained by DTBA and ASBA

5 Conclusion

In this paper, two meta-heuristic algorithms, including the driving training-based algorithm (DTBA) and the average and subtract based algorithm (ASBA), are applied to determine the optimal solution for different optimization problems with different degrees of complexity. The results obtained by these algorithms are discussed, analyzed, and fairly compared on specific aspects, including the minimum, average, and maximum convergences. Particularly, on the first optimization problem, DTBA only needs over 40 iterations to reach the optimal value, while ASBA must go through over 65 iterations to obtain the same value. On the second optimization problem, the superiority of DTBA over ASBA is undeniable in all convergences. On the third and fourth optimization problems, DTBA still maintains its high efficiency over ASBA by achieving the optimal fitness value with fewer iterations than ASBA. Through the validation conducted on

each aspect of each considered optimization problem, DTBA is completely superior to ASBA in almost all comparison aspects. Hence, we highly suggest using DTBO to deal with such optimization problems. Besides, this study also has several drawbacks that need to be improved for better quality in the future version, as follows: 1) This study only validates the performance of the applied methods by using the theoretical optimization problem; there are no real-life optimization problems that have been solved in this paper. 2) The quantity of optimization problems tested stops at four, which is quite little to judge the real performance of the applied methods. 3) The algorithms applied in this study are the original version, and there are no modifications implemented on them for better performance.

Compliance with ethical standards

Acknowledgments

We acknowledge Ly Tu Trong College for supporting this study.

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