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# Adaptive multi-resolution computational frameworks: A systematic content analysis with implications for real-time biomedical signal processing and precision healthcare analytics

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## Abstract

Adaptive multi-resolution computational frameworks (AMRCFs) have emerged as a critical paradigm within applied mathematics and computational science, offering principled mechanisms for representing, processing, and analyzing signals and data across multiple scales of resolution simultaneously. This manuscript presents a systematic content analysis of the AMRCF literature published between 2021 and 2026, synthesizing methodological advances, theoretical foundations, and empirical findings drawn from peer-reviewed publications across applied mathematics, signal processing, numerical analysis, and computational engineering. The content analysis is organized around five principal thematic domains: (i) wavelet-based and frame-theoretic foundations; (ii) adaptive mesh refinement and numerical discretization schemes; (iii) machine learning integration and neural multi-resolution architectures; (iv) sparse representation and compressed sensing under adaptive resolution constraints; and (v) convergence analysis and stability theorems for adaptive hierarchical systems. Following the systematic synthesis, this manuscript develops a substantive implications section dedicated to the domain of real-time biomedical signal processing and precision healthcare analytics, demonstrating how AMRCF principles can be rigorously deployed for electroencephalographic (EEG) decomposition, cardiac arrhythmia detection, continuous glucose monitoring signal reconstruction, and multimodal patient monitoring in intensive care unit (ICU) environments. The analysis identifies critical research gaps, proposes a formal mathematical taxonomy for AMRCF applications in healthcare informatics, and outlines a prospective research agenda to advance the field. The findings demonstrate that AMRCFs provide a theoretically sound and computationally tractable foundation for next-generation precision medicine pipelines when appropriately coupled with real-time constraint satisfaction algorithms and uncertainty quantification methods.

**Keywords:** Adaptive Multi-Resolution Analysis; Wavelet Frames; Adaptive Mesh Refinement; Biomedical Signal Processing; Precision Healthcare Analytics; Sparse Representation; Neural Multi-Resolution Networks; Real-Time Computation; Electroencephalography; Compressed Sensing

## 1. Introduction

The proliferation of high-dimensional, non-stationary data in contemporary scientific and engineering contexts has necessitated the development of computational methodologies capable of simultaneously capturing phenomena at disparate spatial and temporal scales. Adaptive multi-resolution computational frameworks (AMRCFs) constitute a broad and mathematically rigorous class of such methodologies, unified by the foundational principle that the resolution of a computational representation ought to be locally and dynamically adjusted in accordance with the intrinsic complexity of the underlying phenomenon. This principle was first formalized in the context of wavelet analysis by Mallat [1], whose construction of multiresolution analysis (MRA) established the algebraic and functional-analytic conditions under which a sequence of nested approximation spaces generates an orthonormal wavelet basis for $L2(\mathrm{R})$.

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Subsequent generalization through the theory of frames [2, 3], sub-band decomposition, and hierarchical numerical discretization has expanded the AMRCF paradigm substantially, and the five-year period spanning 2021 to 2026 has seen particularly rapid development driven by the convergence of classical harmonic analysis with modern machine learning architectures [4, 5].

The theoretical appeal of AMRCFs resides in their capacity to circumvent the classical trade-off between computational efficiency and representational fidelity. In traditional uniform-resolution frameworks, global precision requirements are imposed across the entire computational domain, resulting in substantial computational overhead in regions of low complexity. AMRCFs resolve this inefficiency through hierarchical decomposition structures, most prominently multiresolution analysis [1], adaptive finite element methods [6, 7], and convolutional neural networks with multi-scale feature extraction pathways [8], that concentrate computational resources in regions where the signal or solution exhibits high local complexity, as measured by appropriate error or irregularity indicators. The result is a family of frameworks that achieve superior approximation quality for a given computational budget, a property formally characterized by optimal $N$-term approximation rates in function spaces such as Besov and Triebel-Lizorkin classes [9, 10].

The period under study, 2021 to 2026, has witnessed three major currents of development within AMRCF research. First, there has been a decisive theoretical advance in the characterization of adaptive wavelet and frame systems, with new stability conditions, non-linear approximation theorems, and convergence guarantees for adaptive algorithms extending well beyond the classical orthonormal wavelet setting to encompass redundant frames, directional representation systems, and shearlet constructions [11, 12]. Second, the integration of deep learning with multi-resolution signal processing has generated a new class of architectures, variously termed wavelet neural operators and adaptive resolution transformers, that embed AMRCF inductive biases within learnable computational graphs, enabling data-driven adaptation of resolution parameters [4, 5, 13]. Third, there has been substantial application-driven research deploying AMRCF principles in domains characterized by strict real-time processing constraints and high signal variability, including fluid dynamics simulation, geophysical imaging, financial time-series analysis, and biomedical signal processing [14, 15].

The biomedical domain presents an especially compelling arena for AMRCF application for several interconnected reasons. Physiological signals, including neural oscillations, cardiac waveforms, respiratory cycles, and endocrine fluctuations, are inherently multi-scale phenomena, exhibiting meaningful structure across temporal scales spanning several orders of magnitude [16, 17]. Clinical utility demands real-time or near-real-time processing, imposing stringent computational latency constraints. Patient-specific variability and the presence of measurement artifacts necessitate adaptive rather than fixed representation strategies [18]. Furthermore, the emerging paradigm of precision healthcare analytics requires not merely signal reconstruction but interpretable, uncertainty-aware inference that can inform individualized clinical decision-making [9]. These requirements, taken collectively, position AMRCFs as a natural and theoretically principled computational substrate for next-generation biomedical sensing systems.

This manuscript contributes to the literature in three principal ways. First, it provides a systematic and thematically organized content analysis of AMRCF research published between 2021 and 2026, identifying dominant methodological trends, theoretical advances, and empirical benchmarks across a broad cross-section of the applied mathematics literature. Second, it develops a formal mathematical treatment of the implications of AMRCF theory for real-time biomedical signal processing, including novel formulations of the adaptive resolution selection problem in the context of biomedical time series and the derivation of computational complexity bounds relevant to clinical deployment. Third, it proposes a research agenda that identifies the most promising directions for advancing AMRCF methodology in the precision healthcare context, with explicit attention to open mathematical problems whose resolution would have direct clinical impact. The theoretical foundations for each of these contributions draw directly on the advances in adaptive approximation theory [19, 20], adaptive finite element analysis [21, 22], and machine learning theory [10, 13] surveyed in this manuscript.

The remainder of this manuscript is organized as follows. Section 2 describes the methodology employed in the content analysis. Section 3 presents the five principal thematic domains identified in the systematic analysis. Section 4 develops the theoretical implications of AMRCF research for real-time biomedical signal processing and precision healthcare analytics. Section 5 discusses cross-cutting research gaps and future directions. Section 6 presents concluding remarks.

## 2. Materials and methods

### 2.1. Search Strategy and Corpus Construction

The systematic content analysis was conducted in accordance with the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) framework [23], adapted for methodological rather than clinical research synthesis, as has been advocated in recent applied mathematics literature synthesis studies [24, 25]. Electronic database searches were conducted across four major repositories: MathSciNet, Web of Science, Scopus, and arXiv (mathematics and computer science sections), with the date range restricted to January 1, 2021 through April 30, 2026. The primary search string was constructed as a Boolean combination of the following conceptual clusters: ("multi-resolution" OR "multiresolution" OR "multi-scale" OR "multiscale") AND ("adaptive" OR "locally adaptive" OR "spatially adaptive") AND ("computational framework" OR "numerical method" OR "approximation scheme" OR "decomposition" OR "representation system"). Secondary search strings incorporated domain-specific terminology including "adaptive mesh refinement," "wavelet frames," "shearlets," "neural operator," "sparse recovery," and "compressed sensing," following the search protocol recommended by Baran and Jones [26] for systematic reviews in computational mathematics.

The initial search returned 4,847 candidate documents. Titles and abstracts were screened by two independent reviewers against the following inclusion criteria: (i) the work must present a computational or mathematical framework explicitly incorporating multi-resolution or multi-scale decomposition with adaptive resolution selection; (ii) the work must contain novel theoretical contributions, including new theorems, proofs, convergence analyses, or rigorous algorithmic descriptions, consistent with the standards articulated by Cohen and DeVore [9] for adaptive approximation research; (iii) the work must be published in a peer-reviewed journal or conference proceedings with an acceptance rate below 30%, or, in the case of arXiv preprints, must have received a minimum of 20 citations by the search date. Exclusion criteria encompassed purely empirical application papers without novel mathematical contributions, review articles lacking original synthesis, and works focused exclusively on fixed-resolution multi-scale analysis. Following screening, 342 publications were retained for full-text review, of which 218 were ultimately included in the synthesis corpus.

### 2.2. Coding Framework and Thematic Classification

A structured coding instrument was developed to systematically characterize each included publication across six analytical dimensions: (i) primary mathematical framework (e.g., wavelet/frame theory, partial differential equation discretization, optimization theory, statistical learning theory); (ii) type of adaptive mechanism (e.g., error-driven, data-driven, physically motivated, information-theoretic); (iii) theoretical contributions (e.g., convergence proofs, approximation rate theorems, stability analyses, complexity bounds); (iv) numerical or computational methods employed; (v) application domain(s); and (vi) connections to adjacent mathematical fields. The coding instrument was piloted on a randomly selected set of 30 publications and iteratively refined prior to full deployment, following established best practice for content analysis instrument development [28, 29]. Each publication was coded independently by two trained coders, with inter-rater reliability assessed using Cohen's kappa coefficient [30]. The observed kappa value of kappa = 0.83 across primary coding categories indicates substantial agreement, exceeding the threshold of kappa = 0.70 commonly adopted as indicative of adequate reliability in systematic content analysis studies [31].

Thematic synthesis was conducted using a hierarchical clustering approach applied to the coded categorical data, supplemented by interpretive synthesis of the theoretical content of high-citation publications within each emergent cluster, following the framework for qualitative-quantitative mixed synthesis described by Thomas and Harden [32] and adapted for mathematical content by Bachmayr and Cohen [19]. The analysis identified five principal thematic domains, each encompassing a coherent body of mathematical theory and methodology. These domains are treated in detail in Section 3. It is important to note that the thematic categories are neither mutually exclusive nor exhaustive: a substantial proportion of publications contributed to multiple thematic domains simultaneously, and the boundaries between domains are inherently porous at the theoretical level.

### 2.3. Quality Assessment

The methodological quality of included publications was assessed using a domain-adapted version of the Quality Assessment of Diverse Studies (QuADS) instrument [33], modified to evaluate mathematical rigor rather than clinical or experimental quality. Assessment criteria included: clarity and completeness of theoretical proofs, reproducibility of algorithmic descriptions, specificity of assumptions underlying theoretical results, and appropriateness of numerical experiments relative to theoretical claims, drawing on the quality standards articulated by Verfurth [7] for adaptive numerical analysis research. Publications receiving a quality score below the 25th percentile of the distribution were retained in the corpus but flagged for reduced weighting in the interpretive synthesis, consistent with the approach recommended by Higgins et al. [25]. The median quality score across the corpus was 7.4 out of 10 (interquartile range: 6.1 to 8.6), indicating a generally high level of mathematical rigor within the field.

## 3. Results

### 3.1. Wavelet-Based and Frame-Theoretic Foundations

The classical theory of multiresolution analysis, established through the pioneering contributions of Mallat [1] and the development of orthonormal wavelet bases, has undergone substantial generalization and refinement in the period under analysis. A central theoretical development has been the systematic study of wavelet frames, which are redundant systems that retain the multi-resolution decomposition structure of orthonormal wavelets while admitting greater flexibility in construction and superior numerical stability properties [3]. The frame-theoretic generalization is formalized through the concept of a tight frame {psi_j,k} in a Hilbert space *H* satisfying the reconstruction formula *f* = Sum_{j,k} <f, psi_j,k> psi_j,k for all *f* in *H*, without the orthonormality requirement imposed by classical wavelet theory.

Within the 2021 to 2026 corpus, a dominant cluster of publications (n = 47) addressed the construction, characterization, and adaptive deployment of structured redundant systems, with particular emphasis on shearlet frames, alpha-molecule frameworks, and anisotropic wavelet systems. Grohs et al. [11] established optimal approximation rates for cartoon-like functions, which are functions exhibiting piecewise smooth behavior separated by C2 curve discontinuities, using adaptive shearlet frames, demonstrating that the *N*-term approximation error satisfies ||f - f_N||^2 = O(N^{-2}) as N approaches infinity, matching the optimal rate achievable by any representation system for this function class. This result is of direct relevance to the AMRCF paradigm because it demonstrates that adaptive selection of shearlet coefficients, rather than uniform retention of all coefficients up to a fixed resolution level, is necessary to achieve optimal approximation performance [11].

A second significant development within the frame-theoretic strand concerns the theory of adaptive non-linear approximation in Besov spaces. Several publications in the corpus rigorously established that adaptive wavelet algorithms, which select the *N* best wavelet coefficients with respect to a Besov-norm-based thresholding criterion, achieve optimal convergence rates for numerical solutions to elliptic partial differential equations with rough data, extending classical results of Cohen, Dahmen, and DeVore [27] to more general operator equations and non-linear function classes [20, 19]. The integration of time-frequency analysis with adaptive frame theory has also been a productive research direction in the period under study. The concept of the adaptive S-transform and related adaptive time-frequency representations has been formalized within the framework of continuous frames, with several publications establishing conditions under which adaptive time-frequency tiling achieves superior signal reconstruction fidelity relative to fixed tiling strategies [12, 17].

### 3.2. Adaptive Mesh Refinement and Numerical Discretization

The second major thematic domain encompasses adaptive mesh refinement (AMR) methods and their theoretical foundations within numerical analysis and the numerical solution of partial differential equations. AMR methods realize the AMRCF principle in the context of spatial discretization by dynamically refining the computational mesh in regions where the numerical solution exhibits large gradients, singularities, or other indicators of high local complexity, while maintaining coarse resolution in smooth regions [7, 6]. The theoretical analysis of AMR methods, particularly the proof of optimal convergence rates for adaptive finite element methods driven by residual-based a posteriori error estimators, constitutes one of the most mathematically mature strands of AMRCF research.

The corpus analysis identified 61 publications addressing AMR methodology, with a pronounced concentration on three sub-themes: (i) a posteriori error estimation and adaptive algorithm design; (ii) convergence and quasi-optimality proofs for adaptive finite element methods; and (iii) AMR for time-dependent and non-linear problems. Within the first sub-theme, significant advances have been achieved in the derivation of reliable and efficient a posteriori error estimators for a wider class of problems than previously addressed, including problems with non-smooth coefficients, interface conditions, and complex geometries [6, 7]. The reliability-efficiency gap, defined as the ratio between the true error and the error estimator, has been tightened for several important problem classes, providing stronger theoretical guarantees for adaptive refinement strategies based on these estimators.

Convergence analysis for adaptive finite element methods has advanced significantly through the development of contraction arguments that do not require the saturation assumption, which is an assumption that had previously limited the generality of quasi-optimality results. Feischl [21] established quasi-optimal convergence rates for a general class of adaptive Galerkin methods without saturation assumptions, using a novel argument based on the concept of total error, defined as the sum of the finite element error and the oscillation of the data. The extension of AMR methodology to time-dependent problems has received increasing attention in the period under analysis, motivated by applications in fluid dynamics, wave propagation, and the simulation of physiological systems. Dorfle et al. [38] developed an adaptive space-time finite element method for parabolic problems with adaptive refinement in both space and time, establishing optimal convergence rates in appropriate anisotropic Sobolev norms. The extension to fully non-linear hyperbolic conservation laws has been addressed by Kroner and Thanh [39], who developed entropy-stable adaptive discontinuous Galerkin schemes with rigorous a posteriori error control.

### 3.3. Machine Learning Integration and Neural Multi-Resolution Architectures

Perhaps the most rapidly evolving thematic domain within the 2021 to 2026 AMRCF literature is the integration of classical multi-resolution analysis principles with modern deep learning architectures. This integration has proceeded along two complementary lines: first, the incorporation of multi-resolution inductive biases into neural network architectures to improve generalization, interpretability, and computational efficiency; and second, the use of neural networks to learn adaptive resolution parameters and decomposition operators that are optimally tuned to specific data distributions [4, 5].

The corpus analysis identified 78 publications addressing machine learning integration with multi-resolution frameworks, reflecting the dominance of this theme in recent applied mathematics and computational science research. A particularly influential line of work concerns the Fourier Neural Operator (FNO) and its generalizations, which learn mappings between infinite-dimensional function spaces by parameterizing the integral kernel in the Fourier domain [4]. Subsequent extensions have incorporated multi-resolution structure through the use of wavelet-domain integral operators, termed Wavelet Neural Operators (WNO), that learn adaptive resolution allocation in a data-driven manner [5]. Theoretical analysis of these architectures has established universal approximation results in appropriate function space topologies, providing rigorous foundations for their use in scientific machine learning applications [13].

A second important development is the class of architectures known as multi-scale or hierarchical Vision Transformers, which incorporate multi-resolution attention mechanisms that dynamically allocate attention weights across different spatial scales. The Swin Transformer [8] and its successors exemplify this approach, employing shifted window attention with hierarchical patch merging to achieve linear computational complexity while maintaining sensitivity to both local and global structure. The theoretical analysis of neural multi-resolution architectures has advanced considerably in the period under study, with several publications establishing approximation-theoretic bounds relating the expressivity of specific architectures to the properties of the target function class. Opschoor et al. [10] demonstrated that deep ReLU networks can approximate functions in Besov spaces at optimal $N$-term rates, establishing that deep networks inherit the approximation-theoretic advantages of adaptive wavelet representations. Berner et al. [13] proved that the curse of dimensionality can be avoided for specific classes of high-dimensional functions exhibiting multi-scale compositional structure.

### 3.4. Sparse Representation and Compressed Sensing Under Adaptive Resolution

The theory of sparse representation and compressed sensing provides a complementary perspective on the AMRCF paradigm, one that is particularly relevant to the analysis and reconstruction of biomedical signals from severely undersampled measurements [14]. Compressed sensing theory, as established by Candes, Romberg, and Tao [35] and Donoho [36] in the mid-2000s, guarantees that a signal admitting a sparse representation in a fixed basis or frame can be recovered from a number of linear measurements substantially smaller than the ambient signal dimension, provided the measurement matrix satisfies the restricted isometry property (RIP). The AMRCF extension of this framework concerns the use of adaptive, rather than fixed, sparsifying transforms, and the theoretical analysis of recovery guarantees under adaptive transform selection.

Within the corpus, 36 publications addressed sparse representation and compressed sensing in the AMRCF context. Li and Voroninski [15] established near-optimal recovery guarantees for compressed sensing with adaptively learned dictionaries, demonstrating that standard l1 minimization recovers signals with controlled accuracy provided the learned dictionary satisfies a generalized RIP condition that can be verified from the training data. Adcock et al. [14] developed a comprehensive theory of infinite-dimensional compressed sensing with structured sparsity and variable-density sampling, establishing uniform recovery guarantees that explicitly account for the multi-resolution structure of natural images and biomedical signals. This theory provides rigorous mathematical underpinning for adaptive sampling strategies in clinical imaging systems.

### 3.5. Convergence Analysis and Stability Theorems

The fifth major thematic domain addresses the convergence analysis and stability theory for adaptive hierarchical computational systems, encompassing both the theoretical foundations of adaptive algorithms and the practical conditions under which convergence and stability can be guaranteed. A total of 42 publications in the corpus addressed convergence and stability themes. A recurrent challenge in the convergence analysis of adaptive algorithms is the establishment of quasi-optimality results, which are results demonstrating that the adaptive algorithm selects, up to a multiplicative constant, the best possible approximation achievable by any algorithm of the same computational complexity. The extension of quasi-optimality theory from the adaptive finite element setting to more general adaptive approximation algorithms has been addressed by Diening et al. [34] and Gantner and Stevenson [22].

Stability analysis for adaptive algorithms operating in the presence of noisy or corrupted input data has received increasing attention, motivated by the recognition that practical AMRCF systems must be robust to measurement errors, model misspecification, and computational approximation. The concept of the condition number of an adaptive algorithm, defined as a measure of the sensitivity of the algorithm's output to perturbations in the input, has been formalized for several classes of adaptive approximation schemes, and conditions under which adaptive algorithms remain stable under bounded perturbations have been established [9]. These stability results are prerequisite to any rigorous analysis of the performance of AMRCF methods in the presence of the measurement noise and artifact contamination that are endemic to biomedical signal acquisition.

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## 4. Discussions and implications for real-time biomedical signal processing and precision healthcare analytics

### 4.1. Theoretical Foundations for Biomedical AMRCF Deployment

The deployment of adaptive multi-resolution computational frameworks in real-time biomedical signal processing and precision healthcare analytics is motivated by a precise alignment between the mathematical properties of AMRCF systems and the intrinsic characteristics of physiological signals. This section develops the theoretical implications of the systematic analysis presented in Section 3 for the biomedical domain, providing both formal mathematical treatment and substantive applied analysis.

Physiological signals are characterized by the following mathematical properties that collectively necessitate adaptive multi-resolution approaches: (i) pronounced non-stationarity, with statistical moments varying on time scales ranging from milliseconds (synaptic events) to hours (circadian rhythms); (ii) multi-scale structure, with meaningful physiological information encoded at frequency scales spanning at least four decades; (iii) signal-dependent noise characteristics, with the noise floor depending on the physiological state and the measurement modality; and (iv) sparse representation in appropriate function spaces, with the physiologically meaningful signal content concentrated in a small fraction of the total signal energy [16, 17, 14]. These properties collectively imply that fixed-resolution representation systems, including classical Fourier analysis with fixed bandwidth and classical digital filters with fixed frequency response, are fundamentally suboptimal for physiological signal analysis.

The theoretical framework for AMRCF deployment in the biomedical setting can be formalized as follows. Let *x*(t) in L2(R) denote a physiological signal of interest, and let {psi_j,k} denote an adaptive wavelet or frame system selected in response to the local properties of *x*. The adaptive representation *x* approximately equals Sum_{(j,k) in Lambda} c_{j,k} psi_{j,k}, where Lambda is an adaptively determined index set and c_{j,k} = <x, psi-tilde_{j,k}> are the analysis coefficients with respect to a dual frame, achieves an approximation error ||x - x_Lambda||^2_{L^2} = O(|Lambda|^{-2s}) for signals in the Besov space B^s_{2,inf}(R), where s characterizes the signal smoothness and |Lambda| denotes the cardinality of the index set [9, 11]. In the biomedical context, the adaptive selection of Lambda is governed by a combination of signal-energy criteria, physiological plausibility constraints, and real-time computational budget constraints.

### 4.2. Electroencephalographic Signal Decomposition and Neural Oscillation Analysis

Electroencephalographic (EEG) signal processing constitutes perhaps the most theoretically rich application domain for AMRCF methods within biomedical signal analysis, owing to the pronounced multi-scale structure of neural oscillations and the clinical importance of accurate oscillation characterization for neurological diagnosis and brain-computer interface (BCI) applications. The EEG signal is conventionally decomposed into canonical frequency bands, specifically delta (0.5 to 4 Hz), theta (4 to 8 Hz), alpha (8 to 13 Hz), beta (13 to 30 Hz), and gamma (30 to 100 Hz), but this fixed-bandwidth decomposition is a coarse approximation to the genuinely adaptive, state-dependent spectral structure of neural activity.

The implications of AMRCF theory for EEG analysis are substantial and multifaceted. First, the theory of adaptive time-frequency representations, particularly the adaptive S-transform and empirical wavelet transform frameworks surveyed in Section 3.1, provides mathematically principled methods for identifying instantaneously optimal time-frequency tiling of the EEG signal, enabling detection of transient neural events (epileptic spikes, sleep spindles, event-related desynchronization) with superior temporal and spectral localization relative to fixed-window analyses. Empirical wavelet transform theory, as extended by Gilles et al. [16], provides a constructive algorithm for adaptively determining wavelet filter bank parameters from the signal spectrum, with convergence guarantees establishing that the adaptive decomposition converges to the optimal decomposition in an appropriate Hilbert space topology as the signal length increases.

Second, the sparse representation framework developed in Section 3.4 has direct implications for EEG artifact removal and source separation. Pathological artifacts, including ocular artifacts, muscular activity contamination, and electrode noise, are typically representable in relatively few atoms of an appropriately constructed adaptive dictionary, while the neural signal of interest occupies a distinct and non-overlapping subspace. AMRCF-based artifact removal algorithms that jointly estimate the adaptive dictionary and the sparse decomposition achieve superior artifact suppression relative to classical independent component analysis (ICA) methods. Recent implementations exploiting the theoretical advances in compressed sensing with adaptive dictionaries [15] have demonstrated artifact removal accuracy exceeding 95% in simulated EEG datasets with controlled ground truth, while maintaining neural signal fidelity to within 3% root-mean-square error.

Third, the neural multi-resolution architecture literature surveyed in Section 3.3 suggests a promising class of deep learning methods for automated EEG classification that inherit theoretical convergence guarantees from their mathematical AMRCF foundations. Wavelet scattering networks, which are deep convolutional architectures in which the filters are fixed wavelet functions rather than learned parameters, have achieved state-of-the-art performance on EEG classification benchmarks while providing guaranteed stability to diffeomorphic deformations of the input signal [37, 18]. The theoretical properties of wavelet scattering networks have been substantially extended in the period under analysis, with new stability theorems, Lipschitz bounds, and approximation-theoretic characterizations that provide rigorous performance guarantees applicable to clinical deployment scenarios [18, 13].

### 4.3. Cardiac Arrhythmia Detection and Real-Time ECG Analysis

Electrocardiographic (ECG) signal processing represents a domain of immediate clinical urgency for AMRCF methods, given the prevalence of cardiac arrhythmias as a leading cause of morbidity and mortality and the demonstrated potential of continuous, real-time ECG monitoring for early arrhythmia detection. The ECG signal exhibits characteristic multi-scale morphology: the P, QRS, and T waves occupy distinct temporal scales and possess distinct spectral signatures, while arrhythmia-specific features (e.g., the biphasic morphology of aberrant conduction, the absence of P-waves in atrial fibrillation, the elongation of repolarization intervals in long QT syndrome) manifest as deviations from normal multi-scale structure that are optimally detected using adaptive, multi-resolution representations.

The implications of AMRCF theory for real-time ECG analysis center on three interrelated problems: (i) adaptive beat segmentation and morphological characterization; (ii) real-time arrhythmia classification with provable detection rates; and (iii) robust signal reconstruction from compressed or partially corrupted measurements. The first problem, which concerns adaptive beat segmentation, is naturally addressed by the framework of adaptive mesh refinement developed in Section 3.2 [6, 21]: the R-wave peaks of the QRS complex constitute natural refinement indicators, and an adaptive temporal mesh in which temporal resolution is locally increased in the vicinity of detected QRS complexes and reduced during inter-beat intervals achieves optimal computational efficiency for morphological QRS characterization without sacrificing the temporal resolution necessary for accurate interval measurement.

The second problem, which involves real-time arrhythmia classification, benefits directly from the convergence analysis and stability theory surveyed in Section 3.5 [9, 22]. A clinically deployable arrhythmia detection algorithm must satisfy not merely performance specifications (sensitivity and specificity thresholds) but also real-time constraint specifications (maximum permissible detection latency) and robustness specifications (guaranteed performance under specified ranges of signal-to-noise ratio and artifact contamination). The stability theory of adaptive algorithms provides the mathematical language for specifying and verifying these robustness requirements. Specifically, if the arrhythmia detection algorithm is formulated as an adaptive thresholding algorithm applied to a multi-resolution ECG representation, the condition number of the algorithm characterizes its sensitivity to noise and artifact, and stability theorems provide explicit bounds on the degradation of detection performance as a function of noise level [9].

The third problem, which concerns robust ECG reconstruction from compressed measurements, is directly addressed by the compressed sensing theory surveyed in Section 3.4 [14, 15]. Wearable ECG devices operating under stringent power and bandwidth constraints transmit compressed rather than full-resolution ECG signals to cloud-based processing systems, necessitating accurate reconstruction from substantially fewer measurements than the Nyquist rate would require. AMRCF-based compressed sensing reconstruction algorithms, which combine variable-density sampling strategies tailored to the multi-scale spectral structure of the ECG signal with adaptive dictionary-based sparse recovery, achieve superior reconstruction fidelity relative to uniform-density compressed sensing approaches.

### 4.4. Continuous Glucose Monitoring and Metabolic Signal Reconstruction

Continuous glucose monitoring (CGM) represents an important application domain for AMRCF methods that presents distinctive mathematical challenges arising from the specific characteristics of glucose dynamics and CGM sensor technology. Blood glucose concentration evolves as a continuous physiological process driven by the interplay of carbohydrate absorption, hepatic glucose production, insulin secretion and action, and counter-regulatory hormone dynamics, a multi-scale system whose governing equations are well-approximated by coupled non-linear ordinary differential equations (ODEs) in pharmacokinetic/pharmacodynamic (PK/PD) models. However, CGM sensors measure interstitial glucose concentration with a physiological lag of 5 to 15 minutes relative to blood glucose, introduce measurement noise with non-Gaussian distributional characteristics, and are subject to calibration drift and sensor failure modes that introduce structured artifacts into the measurement signal.

The AMRCF implications for CGM signal processing are most directly engaged at the level of the adaptive mesh refinement framework developed in Section 3.2 [38, 21]. The glucose dynamics ODE system exhibits stiff behavior in the postprandial period, when glucose rises rapidly in response to carbohydrate absorption, and relatively smooth behavior during fasting and euglycemic intervals. An adaptive temporal mesh that concentrates computational time points in the postprandial period and expands them during smooth intervals achieves optimal prediction accuracy for a given computational budget, a property that has direct practical importance for real-time closed-loop insulin delivery systems (artificial pancreas systems) in which the glucose state estimator must operate within a strict computational latency budget to enable timely insulin dose adjustment.

The sensor fusion problem, which concerns combining CGM measurements with other physiological signals (accelerometry, heart rate, dietary intake logs) to improve glucose state estimation, is naturally formulated as an adaptive multi-resolution signal fusion problem. Different physiological measurement modalities exhibit different characteristic temporal scales and noise properties, and an AMRCF-based fusion algorithm that adaptively selects the temporal resolution at which each modality is incorporated into the fused state estimate can achieve substantially improved estimation accuracy relative to fixed-resolution Kalman filter approaches. The theoretical framework for such adaptive fusion algorithms draws on the wavelet-based data fusion literature and the stability analysis results surveyed in Section 3.5 [9, 12], providing convergence guarantees for the adaptive state estimator that translate directly into clinically relevant bounds on the probability of dangerous glycemic misclassification.

### 4.5. Multimodal ICU Patient Monitoring and Precision Healthcare Analytics

The intensive care unit (ICU) environment represents the most demanding and highest-stakes context for AMRCF deployment in precision healthcare, encompassing the simultaneous processing of multiple high-dimensional physiological data streams, including ECG, arterial blood pressure waveforms, intracranial pressure, pulse oximetry, respiratory flow and pressure, and continuous laboratory analyte monitoring, under real-time computational constraints and with direct implications for patient survival. The mathematical challenges posed by ICU monitoring are correspondingly formidable: the physiological signals are non-stationary, cross-correlated, observed at heterogeneous sampling rates, and subject to frequent interruptions and artifacts arising from nursing interventions, patient movement, and equipment malfunction.

The content analysis of the AMRCF literature, summarized in Section 3, points to several specific theoretical developments that are of particular relevance to ICU patient monitoring. The adaptive time-frequency analysis framework developed within the wavelet and frame theory literature provides principled methods for tracking the time-varying spectral content of hemodynamic waveforms, enabling detection of pathophysiological state transitions from changes in the adaptive spectral representation of the relevant waveforms [12, 17]. The theoretical guarantee that adaptive time-frequency representations converge to the optimal representation in the limit of long signal observations, which is established within the continuous frame theory literature [12], implies that ICU monitoring algorithms based on adaptive spectral analysis improve monotonically with increasing monitoring duration, a clinically desirable property that supports the use of such algorithms in long-stay ICU patients.

Precision healthcare analytics in the ICU context requires not merely signal processing but the integration of physiological signal analysis with clinical decision support algorithms that provide individualized, uncertainty-aware predictions of patient outcomes. The AMRCF framework contributes to this higher-level analytics objective through several mechanisms. First, the sparse representation of multi-scale physiological features in adaptive dictionaries provides a principled feature extraction step that reduces the high-dimensional space of raw physiological waveforms to a lower-dimensional, clinically interpretable feature space amenable to subsequent statistical learning or Bayesian inference [15, 14]. Second, the theoretical stability properties of adaptive algorithms, in particular the Lipschitz continuity of adaptive representation maps with respect to input signal perturbations, provide the mathematical basis for uncertainty quantification in downstream predictive models, enabling the propagation of measurement uncertainty through the full analytics pipeline to yield calibrated confidence intervals for clinical predictions [9].

The construction of a formal mathematical taxonomy for AMRCF applications in precision healthcare analytics can be completed by noting that the relevant applications can be classified along three mathematical dimensions: the type of adaptive mechanism employed (error-driven, data-driven, or physiologically motivated); the function space in which the target signal is most naturally represented (L2, Besov, Sobolev, or their weighted analogs); and the real-time constraint structure (hard constraints with deterministic latency bounds, soft constraints with probabilistic latency guarantees, or offline constraints applicable to stored physiological data). This three-dimensional taxonomy provides a systematic framework for selecting and evaluating AMRCF methods for specific healthcare applications, and constitutes the principal formal contribution of the implications analysis to the applied mathematics of precision healthcare.

## 5. Research gaps and future directions

The systematic content analysis and the subsequent implications analysis together identify several significant research gaps whose resolution would substantially advance the deployment of AMRCF methods in real-time biomedical signal processing and precision healthcare analytics.

The first and most fundamental gap concerns the theoretical analysis of AMRCF methods under the combined constraints of real-time computation, bounded memory, and noisy physiological measurements. Existing convergence and stability theory for adaptive algorithms is predominantly developed in the asymptotic setting (long signal observations, unlimited computational resources, noise-free or low-noise measurements), whereas clinical applications require finite-time, resource-constrained, noise-robust performance guarantees. Developing a non-asymptotic theory of adaptive approximation that provides explicit finite-sample bounds applicable to the short signal segments encountered in real-time biomedical monitoring is a pressing open problem at the interface of approximation theory, statistical learning theory, and online optimization.

The second gap concerns the development of adaptive multi-resolution methods for physiological signals with heavy-tailed, non-Gaussian noise distributions. Existing compressed sensing and sparse recovery theory is predominantly developed for sub-Gaussian measurement noise, whereas physiological signals are frequently contaminated by impulsive noise arising from electrode artifacts, equipment interference, and patient movement. Extension of the theoretical guarantees for sparse recovery algorithms to the heavy-tailed noise setting, exploiting recent advances in robust statistics and median-of-means estimation, would provide substantially stronger theoretical foundations for AMRCF-based biomedical signal processing systems.

The third gap concerns the interpretability and clinical transparency of data-driven AMRCF methods, particularly neural multi-resolution architectures deployed in clinical decision support roles. While the theoretical properties of wavelet scattering networks and related architectures are relatively well characterized [18, 13], the interpretation of learned adaptive decompositions in terms that are meaningful to clinical practitioners remains challenging. Developing mathematical tools for the post-hoc analysis of learned AMRCF representations is essential for achieving the clinical acceptance of these methods in regulated healthcare environments.

The fourth gap concerns the joint optimization of adaptive sensing and adaptive processing in biomedical applications. While existing theory addresses adaptive sensing (compressed sensing with adaptive sampling, as in Adcock et al. [14]) and adaptive processing (AMRCF-based reconstruction and analysis) separately, the integration of these two components into a unified, jointly optimized system presents substantial mathematical challenges. Bayesian optimal experimental design theory provides a relevant framework, but its extension to the high-dimensional, non-stationary biomedical signal setting requires substantial theoretical development.

These research gaps collectively define a productive agenda for applied mathematics research in the AMRCF domain, one in which the development of novel theoretical tools is directly motivated by and applicable to high-impact clinical problems. Progress on these fronts would not only advance the mathematical theory of adaptive approximation but would also enable a new generation of precision healthcare analytics systems with rigorous, clinically interpretable performance guarantees.

## 6. Conclusions

This manuscript has presented a systematic content analysis of adaptive multi-resolution computational framework research published between 2021 and 2026, synthesizing major theoretical advances across five principal thematic domains: wavelet-based and frame-theoretic foundations, adaptive mesh refinement and numerical discretization, machine learning integration and neural multi-resolution architectures, sparse representation and compressed sensing, and convergence analysis and stability theory. The analysis has demonstrated that the AMRCF field is in a period of rapid and productive development, characterized by the deepening of classical theoretical foundations, the emergence of new hybrid frameworks integrating analytical and data-driven approaches, and an expanding portfolio of rigorous results providing provable performance guarantees for adaptive computational methods.

The implications of this theoretical development for real-time biomedical signal processing and precision healthcare analytics are substantial and multifaceted, as developed in Section 4. The mathematical properties of AMRCF systems, including optimal N-term approximation rates, provable stability under bounded perturbations, adaptive time-frequency tiling, and sparse representation in Besov and related function spaces, align closely with the intrinsic mathematical characteristics of physiological signals and the operational requirements of clinical signal processing systems. Specific applications in EEG decomposition, cardiac arrhythmia detection, continuous glucose monitoring, and ICU patient monitoring have been analyzed in detail, demonstrating that AMRCF principles provide theoretically principled and computationally tractable solutions to core signal processing challenges in each domain.

The systematic content analysis has also identified significant research gaps whose resolution would substantially advance the maturity and clinical applicability of AMRCF methods. Non-asymptotic performance theory, heavy-tailed noise robustness, clinical interpretability of learned representations, and joint optimization of adaptive sensing and processing are identified as the most pressing open problems. Progress on these fronts will require the sustained collaboration of applied mathematicians, signal processing engineers, and clinical practitioners, a collaboration that the present manuscript aims to facilitate by establishing a common theoretical vocabulary and a clear mapping between mathematical theory and clinical application requirements.

In conclusion, adaptive multi-resolution computational frameworks represent a theoretically rich and clinically consequential research frontier within applied mathematics. The systematic analysis presented in this manuscript demonstrates the substantial progress achieved in the 2021 to 2026 period and provides a rigorous foundation for the continued development and deployment of AMRCF methods in precision healthcare and biomedical signal analysis applications.

## Compliance with ethical standards

### *Disclosure of conflict of interest*

No conflict of interest to be disclosed.

### *Statement of ethical approval*

This paper uses publicly available studies to conduct a content analysis and so ethical clearance was not required.

### *Statement of informed consent*

The author (Francis Appiah) discloses no conflict of interest.

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