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(Review Article)

On primitivity and regularity of wreath product groups of degree 5p that are not p-groups using numerical approach

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Abstract

Let p be an odd prime number. This work applies some group concepts to construct the Wreath Product of two permutation groups of prime degrees. We used numerical approach to investigate and determine the primitive and regular nature of the constructed Wreath Product Group of degree 5p. We apply Computational Group Theory (GAP) to facilitate as well as validate our results.

Keywords: Groups; Wreath Product Group; Primitivity; Regularity; Degree 5p; Permutation group

1 Introduction

Group theory plays great roles in every branch of mathematics where symmetry is studied. Every symmetrical object has to do with group. It is due to this association that groups arose in different area like Aeronautical Engineering, Crystallography, Biology, Chemistry, Sociology, etc.

Recently, wreath product groups have been used to explore some useful characteristics of finite groups in connection with permutation designs and construction of lattices [1] as well as in the study of interconnection networks [2].

2 Preliminaries

We present some basic concepts and results that will be applied further:

2.1 Definition of some terms

2.1.1 Stabilizer

A kind of dual role is played by the set of elements in G which fix a specified point α. This is called the stabilizer of α in G and is denoted by $G_\alpha := \{x \in G | \alpha^x = \alpha\}$.

2.1.2 Wreath Products

The wreath product of C by D denoted by W=Cwr D is the semi-direct product of P by D, so that, $W = \{(f,d) \mid f \in P, d \in D\}$ with multiplication in W defines as $(f_{1,}d_1)(f_2d_2) = ((f_1f_2^{d_1^{-1}}),(d_1d_2))$ for all $f_1f_2 \in P$ and $d_1,d_2 \in D$.. Henceforth, we write fd instead of (fd) for elements of W.

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Note; We wish to henceforth notice that

If C and D are finite groups then a wreath product W determines by an action of D on a finite set is a finite group of order $|W| = |C|^{\Delta}.|D|$.

P is normal subgroup of W and D is a subgroup of W.

The action of W on $\Gamma\ x\ \Delta$ is given by $(\alpha, \beta\ )fd = (\alpha f(\beta), \beta d)$ where $\alpha \in \Gamma$ and$\beta \in \Delta$.

2.1.3 Transitive Groups

A group G acting on a set Ω is said to be transitive on Ω if it has one orbit and so $\alpha^G = \Omega$ for all $\alpha \in \Omega$. Equivalently, G is transitive iff or every pair of point $\alpha, \delta \in \Omega$ there exists $g \in \Omega$ such that $\alpha^g = \beta$. A group which is not transitive is called intransitive.

If $|\Omega| \geq 2$, we say that the action of G on Ω is doubly transitive iff for any $\alpha_1, \alpha_2 \in \Omega$ such that $\alpha_1 \neq \alpha_2$ and $\beta_1, \beta_2 \in \Omega$ such that $\beta_1, \neq \beta_2$ there exist $g \in G$ such that $\alpha_1^g = \beta_1, \alpha_2^g = \beta_2$.

The group G is said to be k-transitive (or k-fold transitive) on Ω iff or any sequences $\alpha_1, \alpha_2, \ldots, \alpha_k$ such that $\alpha_i \neq \alpha_j$ when $i \neq j$ and $\beta_1, \beta_2, \ldots, \beta_k$ such that $\beta_i \neq \beta_j$ when $i \neq j$ of k element on Ω, there exists $g \in G$ such that $\alpha_1^g = \beta_1\ for\ 1 \leq i \leq k$

Thus,

$$G_1 = \{(1), (12), (13), (23), (123), (132)\} \text{ is transitive and}$$

$$G_2 = \{(1), (12), (34), (12)(34)\} \text{ is intransitive.}$$

2.1.4 Imprimitivity

A subset Δ of Ω is said to be a set of imprimitivity for the action of G on Ω, if for each $g \in G$, either $\Delta^g = \Delta$ or Δ^g and Δ are disjoint. In particular, Ω itself, the 1-element subsets of Ω and the empty set are obviously sets of imprimitivity which are called trivial set of imprimitivity.

Example

The group of symmetry $D_4 = \{(1), (1234), (13)(24)(1432), (13), (24), (12)(34), (14)(23)\}$,of the square with vertices 1,2,3,4 is not primitive. For take $G_1 = \{(1), (24)\}$ = reflection in the line joining vertices 1 and 3 = stabilizer of the point 1, and $H = \{(1), (24), (13)$ = reflection in m the line joining vertices 2 and 4, $(13)(24)$ = rotation in 180^0, $H = \{(1), (24), (13), (13)(24)\}$. Then H is a group greater than G_1, but not equal to G.

2.1.5 Primitive

A permutation group G acting on a nonempty set Ω is called primitive if G acts transitively on Ω and G preserves no non trivial partition of Ω. Where non trivial partition means a partition that is not a partition into singleton set or partition into one set Ω. In other word, a group G is said to be primitive on a set Ω if the only sets of imprimitivity are trivial ones otherwise G is imprimitive on Ω, example the group

$$S_3 = \{(1), (12), (13), (23), (123), (132)\} \text{ is primitive. For each } g \in G, \Delta^g = \Delta\ , \Delta^g \cap \Delta \neq \emptyset$$

3 Methodology

We hear present previous results that will be use as reference point in other to achieve our desired results.

3.1 Theorem ([3])

Let G be a transitive permutation group of prime degree on Ω. Then G is primitive.

3.1.1 Proof

Now since G is transitive, it permutes the sets of imprimitivity bodily and all the sets have the same size. But $\Omega = \cup |\Omega_i|$, Ω_i being the sets of imprimitivity. As $|\Omega|$ is prime we

Have that either each $|\Omega_i|=1$ or Ω is the set of imprimitivity. So G is primitive.

3.2 Theorem ([3])

Let G be a non-trivial transitive permutation group on Ω. Then G is primitive iff G_α, $(\alpha \in \Omega)$ is a maximal subgroup of G or equivalently, G is imprimitivity if and only if there is a subgroup H of G properly lying between G_α, $(\alpha \in \Omega)$ and G.

3.2.1 Proof:

Suppose G is imprimitive and ψ a non-trivial subset of imprimitivity of G.

$$\text{Let } H = \{g \in G | \psi^g = \psi\}.$$

Clearly H is a subgroup of G and a proper subgroup of G because $\psi \subset \Omega$ and G is transitive.

Now choose $\alpha \in \psi$. If $g \in G$ then $\alpha^g = \alpha$, showing that $\alpha \in \psi \cap \psi^g$ and so $\psi = \psi^g$.

Hence $G \leq H$.

Hence $G_\alpha \leq H \leq G$.

Since $|\psi| = 1$, choose $\beta \in \psi$ *such that* $\beta \neq \alpha$. By transitivity of G, there exist some $h \in G$ with $\alpha^h = \beta$ *so that* $h \in G_\alpha$. Now $\beta \in \psi \cap \psi^h$ so $\psi = \psi^g$ and $h \in H - G_h$. Thus, $H \neq G_\alpha$ Hence G_α is not a maximal subgroup.

Conversely, suppose that $G_\alpha \leq H \leq G$ for some subgroup H.

Let $\psi = \alpha^H$. Since $H > G_\alpha$, $|\psi| \neq 1$.

Now If $\psi = \Omega$, then H is transitive on Ω and hence $\Omega = |G: G_\alpha| = |H: G_\alpha$ showing that H = G, a contradiction. Hence, $\psi = \Omega\psi = \Omega$. Now we shall show that ψ is a subset of imprimitivity of G.

Let $h \in G$ and $\beta \in \psi \cap \psi^g$ then $\beta = a^h = a^{hg}$ for some $h,h' \in H$.

Hence $\alpha_{hgh^{-1}} = \alpha$. So $hgh^{-1} \in G_\alpha < H$.

Thus $\psi = \psi^g$. Hence ψ is a non-trivial subset of imprimitivity. So G is imprimitive.

3.3 Theorem Fundamental Counting lemma or Orbit formula ([4])

Let G act on Ω and $\alpha \in \Omega$. If G is finite then $|G| = \left|G_\alpha\right| |\alpha^G|$.

3.3.1 Proof:

We determine the length $|\alpha^G|$ of the α^G, we have that $\alpha^x = \alpha^y$ if and only if $\alpha^{xy^{-1}} = \alpha$ if and only if $\alpha^{xy} \in G_\alpha$ if and only if $G_\alpha x = G_\alpha y$. Thus there is one to one correspondence given by the mapping $G_\alpha x \rightarrow \alpha^x$ between the set of right cosets G_α and the G-orbit α^G in Ω. Accordingly, as G is finite we have that

$$\left|G:G_\alpha\right| = |\alpha^G| \text{ and so } \left|G\right| = |G_\alpha| |\alpha^G|.$$

3.4 Wreath Product ([5])

The Wreath product of two permutation groups C and D denoted by *W = C wr D* is the semi –direct product of P and D so that

$$W = \{(f, d) \mid f \in P, d \in D\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

With multiplication in W defined as

$$(f_1 d_1)(f_2 d_2) = ((f_1, f_2 d_1^{-1})(d_1 d_2)) \text{ for all } f_1, f_2 \in P \text{ and } d_1, d_2 \in D$$

Henceforth we write $f\,d$ instead of *(f,d)* for elements of W

3.5 Theorem ([5])

Let D act on P as $f^{d}(\delta) = f(\delta d^{-1})$ where $f \in P, d \in D$ and $\delta \in \Delta$

Let W be group of all juxtaposed symbols f d with $f \in P, d \in D$ and multiplication given by $(f_1, d_1)(f_2 d_2) = (f_1 f_2 d_1^{-1}) d_1 d_2$ .Then W is a group called the semi-direct product of P by D with the defined action

3.6 Theorem ([5])

Let D act on P as $f^d(\delta) = f(\delta d^{-1}) where f \in P, d \in D and \delta \in \Delta$. Let W be the group of all juxtaposed symbols$f d, with f \in P, d \in D$ and multiplication given by

$(f_1, d_1)(f_{2,} d_2) = \left(f_1 f_2^{d_1^{-1}}. d_1 d_2\right)$. Then W is a group called semi-direct product of P by D with the define action.

3.7 Theorem([6])

Let G be a transitive abelian group. Then, G is regular.

3.7.1 Proof:

Fix $\alpha \in \Omega$. If $\beta \in \Omega$ such that $\exists\, g \in G$ with $\alpha^{G} = \beta$. Now $G_\alpha = G_\alpha^{g} = (G_\alpha)^{g} = g^{-1}(G_\alpha)g = G_\alpha$ (since G is abelian). As α, β are arbitrary, we get that $G_\alpha = 1$ since G is transitive, it is regular.

3.8 Proposition([7])

A transitive group is regular if and only if its order and degree are equal

3.8.1 Proof:

Let G be a regular on Ω. of degree n since$|\alpha^G| = |G|$ and G is transitive Hence $|G| = n$, conversely, by transitivity of G it follows that, $n|G_\alpha| = |G|$.Hence $G_\alpha = 1$, since $|G| = n$ by assumption Hence G is semi-regular, but G is transitive so G is regular

3.9 Proposition ([7])

An intransitive group is irregular if and only if its order and degree are not equal

3.9.1 Proof:

Let G be an irregular group on Ω. of degree n, since $|\alpha^G| \neq |G|$ and G is intransitive Hence $|G| = n$

Conversely by transitivity of G it follows that, $n|G_\alpha| = |G|$.Hence$G_\alpha \neq 1$, since $|G| = n$ by assumption. Hence G is Semi-regular, but G is intransitive so G is irregular.

3.10 Theorem (Orbit-Stabilizer theorem)

Let G be a group acting on a set Ω. Then, for all $\alpha \in \Omega$ $|G_\alpha||\alpha G| = |G|$

3.10.1 Proof:

By proposition 3.2.9 (3), the points αg of the αG are in bijection with the cosets$G_\alpha g. So$ $|\alpha G| = |G{:}\, G_\alpha|$, finally by Langrange's theorem$|G_\alpha||\alpha G| = |G_\alpha||G{:}\, G_\alpha| = |G|$

3.11 Definition 2.1([8])

A transitive action of G on Ω is called regular if $G_{\alpha} = 1$for all ∈ Ω .Equivalently, $g \in G$ fixes no point in Ω.

3.12 Remark ([4])

A group G acting on a set Ω is said to be transitive on Ω if it has only one orbit, and so $\alpha^{G} = \Omega\ for\ all\ \alpha \in \Omega$. Equivalently, G is transitive if for every pair of points $\alpha, \beta \in \Omega$ there exists $x \in G$ such that $\alpha^{G} = \beta$. A group which is not transitive is called intransitive. A group G acting transitively on a set Ω is said to act regularly if $G_{\alpha} = 1$ for each $\alpha \in \Omega$ (equivalently, only the identity fixes any point). The previous theorem then has the following immediate corollary

3.13 Corollary 2.0([8])

Let G act transitively of degree n on a set Ω. Then

- All the stabilizers G_{α},for $\alpha \in \Omega$ are conjugate
- The index $|G{:}G_{\alpha}| = n\ for\ every\ \alpha \in \Omega$
- The action is regular if and only if |G|= n

3.13.1 Proof

Since the action is transitive, by Proposition 3.2.9 (2), all the G_{α} are conjugates. The second two parts follow from the Orbit- Stabilizer and Lagrange's theorem.

Note that, from the first part of the above corollary a transitive group G is regular if there exists $\alpha \in \Omega$ such that $G_{\alpha} = 1$

4 Results and discussions

4.1 Introduction

In this section, we shall be discussing in detail the primitivity and regularity of the Wreath product groups of degrees 5p, and it will be presented in three sections. In Section **4.1** is the introduction, while Section **4.2**, we will present the primitivity and regualrity of the Wreath product group of degree 5p and while section **4.3** Primitivity and Regularity of Wreath Product Group of degree 5p (p=3)

4.2 Primitivity and regularity of Wreath Product Group of Degree 5p.

The following are the main results on the constructed Wreath Product group of degree 5p. (p= 3 $and\ p\ is\ prime$)

4.2.1 Proposition 3.0

Let G and H be transitive permutation groups of prime degrees, there Wreath-Product is neither primitive nor regular.

4.2.2 Proof

Let G and H be the permutation groups of degrees 5 and p respectively. Hence, the Wreath Product $W = GwrH$ and the $|G| = 5\ and\ |H| = p\ so\ |W| = 5^{p}p$, the degree of W is given by $|\alpha^{w}| = |\Omega| = 5p$. Hence, two cases $|W| = 5p^{5}\ or\ |W| = 5^{p}p$

- Case I

$$|W| = 5p^{5}\text{and}|\alpha^{w}| = |\Omega| = 5p$$

From Orbit stabilizer theorem 2.7

$$|\alpha^{W}||W_{\alpha}| = |W|$$

$$|W_{\alpha}| = \frac{|W|}{|\alpha^{W}|}$$

$$|W_{\alpha}| = \frac{5p^{5}}{5p}$$

$$|W_\alpha| = p^{5-1}$$

$$= p^4$$

Since$p \geq 3$, Clearly the stabilizer $|W_\alpha| \neq 1$ Therefore, by Theorem 2.6 and Proposition 2.0 (A transitive group is Regular if and only if its order and degree are equal) also by corollary 2.0 Since the order of W is, $5p^5$ and its degree is 5p, Hence W is irregular. Thus the Wreath Product group is not regular. also since the order and the degree of W are not equal, as clearly stated by Proposition 2.0 and Proposition 2.1 W is not regular and by Theorem 2.0 (Every transitive group of prime degree is primitive) it implies that W is imprimitive, as the degree of W is 5p.

- Case II

$|W| = 5^p p$ and$|\alpha^w| = |\Omega| = 5p$

From Orbit stabilizer theorem 2.7

$$|\alpha^W||W_\alpha| = |W|$$

$$|W_\alpha| = \frac{|W|}{|\alpha^W|}$$

$$|W_\alpha| = \frac{5^p p}{5p}$$

$$|W_\alpha| = 5^{p-1}$$

$$= 5^{p-1}$$

Since$p \geq 3$, Clearly the stabilizer $|W_\alpha| \neq 1$ Therefore, by Theorem 2.6 and Proposition 2.0 (A transitive group is Regular if and only if its order is equal to its degree) also by corollary 2.0 , the order of W is $5^p p$ and its degree is 5p, Hence W is irregular. Thus the Wreath Product group is not regular. also since the order and the degree of W are not equal, as clearly stated by Proposition 2.0 and Proposition 2.1W is not regular and by Theorem 2.0 (Every transitive group of prime degree is primitive) it implies that W is imprimitive, as the degree of W is 5p.

4.3 Primitivity and Regularity of Wreath Product Group of degree 5p (p=3)

Let C be a group of degree 5 and D a group of degree 3 acting on the set $\Omega = \{1,2,3,4,5\}$ and $\Delta = \{6,7,8\}$Respectively Let $P = C^\Delta = \{f: \Delta \to C\}$ with $P = C^\Delta = 5^3$

Then the wreath product$W = CwrD$ of degree = 15 and of order $|W| = \left|C^\Delta\right| \times |D| = 5^3.3 = 375$

We wish to show that W is (i) Imprimitive and(ii) Irregular

4.3.1 Proof

- We follow the procedure as described in theorem 2.4 to obtain the elements of the Wreath Product group W in cyclic form as:

W=[(),(11,12,13,14,15),(11,13,15,12,14),(11,14,12,15,13),(11,15,14,13,12),(6,7,8,9,10),
(6,7,8,9,10)(11,12,13,14,15),(6,7,8,9,10)(11,13,15,12,14),(6,7,8,9,10)(11,14,12,15,13),
(6,7,8,9,10)(11,15,14,13,12),(6,8,10,7,9),(6,8,10,7,9)(11,12,13,14,15),(6,8,10,7,9)(11,13,15,12,14),(6,8,10,7,9)(11,14,12,15,13),(6,8,10,7,9)(11,15,14,13,12),(6,9,7,10,8),(6,9,7,10,8)(11,12,13,14,15),…]

Now$|W| = 375$ and $\Omega = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$ is the set of points of W. It follows by Remark 3.2.6 that W is transitive as the orbit $\alpha^w = \Omega\, \forall \alpha \in \Omega$. Also the stabilizer of the point 1 in W is given by $W_{(1)} = [(11,12,13,14,15), (6,7,8,9,10)]$, which is obviously non-identity proper subgroup of W. We readily see from the subgroups of W that the group (W) has subgroups

H=[(()),[(1,6,11)(2,7,12)(3,8,13)(4,9,14)(5,10,15)],[(1,6,12)(2,7,13)(3,8,14)(4,9,15)(5,10,11)],
[(1,6,13)(2,7,14)(3,8,15)(4,9,11)(5,10,12)],[(1,6,14)(2,7,15)(3,8,11)(4,9,12)(5,10,13)],[
(1,6,15)(2,7,11)(3,8,12)(4,9,13)(5,10,14)],…]

Which is properly lying between $W_{(1)}$ *and* W *that is*, $W_{(1)} < H < W$ hence, W is imprimitive by theorem 2.1 and theorem 2.0. Thus the Wreath Product of W is imprimitive,

- $|W| = 375 = 5^3.3$ and since the degree of W is 6 then $|\Omega| = |\alpha^W| = 15$ by Using (orbit-stabilizer) theorem 2.7

$$|\alpha^W||W_\alpha| = |W|$$

$$|W_\alpha| = \frac{|W|}{|\alpha^W|}$$

$$|W_\alpha| = \frac{375}{15}$$

$$= 5^2$$

Clearly the stabilizer $|W_\alpha| \neq 1$ Therefore, by Theorem 2.6 said W is regular if $|W_\alpha| = 1$ from the above calculation the Stabilizer is equal to 25 which shows that W is not regular and Proposition 2.0(A transitive group is Regular if and only if its order and degree are equal) also by corollary 2.0, Since the order of W is 375 and the degree is 15 Hence W is irregular. Thus the Wreath Product group is not regular.

4.4 Validation of Results

4.4.1 Primitivity and Regularity of Wreath Product Groups of degree 5p (p = 3)

The Group Algorithm and programming version 4.11.1 version

5 Conclusion

This Study showed that the Wreath Product group of degree 5p where p is an odd prime number is

- Imprimitive and
- Irregular

This Study can be extended by considering for further research, one or a combination of two or more of other theoretic properties such as simplicity, nilpotency, solubility etc of same algebraic structure.

Compliance with ethical standards

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Disclosure of conflict of interest

There is no conflict of interest for the Authors

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APPENDIX

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  GAP 4.11.1 of 2021-03-02
 |  GAP  |    https://www.gap-system.org
              Architecture: x86_64-pc-cygwin-default64-kv7
 Configuration:  gmp 6.2.0, GASMAN, readline

gap> # WreathProduct of Permutation Groups of degree 5p
gap> C :=Group((1,2,3,4,5));
Group([ (1,2,3,4,5) ])
gap> D :=Group((6,7,8));
Group([ (6,7,8) ])
gap> W :=WreathProduct (C,D);
Group([ (1,2,3,4,5), (6,7,8,9,10), (11,12,13,14,15), (1,6,11)(2,7,12)(3,8,13)(4,9,14)(5,10,15) ])
gap> Order(W);
375
gap> IsAbelian(W);
false
gap> IsTransitive(W);
true
gap> IsPrimitive(W);
false
gap> IsNilpotent(W);
false
gap> IsRegular(W);
false
gap> W1 :=Stabilizer(W,1);
Group([ (11,12,13,14,15), (6,7,8,9,10) ])
gap> Elements(W);
[ (),(11,12,13,14,15),(11,13,15,12,14),(11,14,12,15,13),(11,15,14,13,12),(6,7,8,9,10),
(6,7,8,9,10)(11,12,13,14,15),(6,7,8,9,10)(11,13,15,12,14),(6,7,8,9,10)(11,14,12,15,13),(6,7,8,9,10)(11,15
,14,13,12),(6,8,10,7,9),(6,8,10,7,9)(11,12,13,14,15),(6,8,10,7,9)(11,13,15,12,14),(6,8,10,7,9)(11,14,12,1
5,13),(6,8,10,7,9)(11,15,14,13,12),(6,9,7,10,8),(6,9,7,10,8)(11,12,13,14,15),(6,9,7,10,8)(11,13,15,12,14)
,(6,9,7,10,8)(11,14,12,15,13), (6,9,7,10,8)(11,15,14,13,12),(6,10,9,8,7),(6,10,9,8,7)(11,12,13,14,15),
(6,10,9,8,7)(11,13,15,12,14),(6,10,9,8,7)(11,14,12,15,13),(6,10,9,8,7)(11,15,14,13,12),
(1,2,3,4,5),(1,2,3,4,5)(11,12,13,14,15),(1,2,3,4,5)(11,13,15,12,14),(1,2,3,4,5)(11,14,12,15,13),
(1,2,3,4,5)(11,15,14,13,12),(1,2,3,4,5)(6,7,8,9,10),(1,2,3,4,5)(6,7,8,9,10)(11,12,13,14,15),
(1,2,3,4,5)(6,7,8,9,10)(11,13,15,12,14),(1,2,3,4,5)(6,7,8,9,10)(11,14,12,15,13),(1,2,3,4,5)(6,7,8,9,10)(1
1,15,14,13,12), (1,2,3,4,5)(6,8,10,7,9),(1,2,3,4,5)(6,8,10,7,9)(11,12,13,14,15), . .. ..
(1,2,3,4,5)(6,8,10,7,9)(11,13,15,12,14),(1,2,3,4,5)(6,8,10,7,9)(11,14,12,15,13),
(1,2,3,4,5)(6,8,10,7,9)(11,15,14,13,12),(1,2,3,4,5)(6,9,7,10,8),
(1,2,3,4,5)(6,9,7,10,8)(11,12,13,14,15),(1,2,3,4,5)(6,9,7,10,8)(11,13,15,12,14),
(1,2,3,4,5)(6,9,7,10,8)(11,14,12,15,13),(1,2,3,4,5)(6,9,7,10,8)(11,15,14,13,12),
(1,2,3,4,5)(6,10,9,8,7),(1,2,3,4,5)(6,10,9,8,7)(11,12,13,14,15),
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,9,13)(5,10,14)]),([[(1,7,12)(2,8,13)(3,9,14)(4,10,15)(5,6,11)]),
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([[(1,7,15)(2,8,11)(3,9,12)(4,10,13)(5,6,14)]),([[(1,7,11)(2,8,12)(3,9,13)(4,10,14)(5,6,15)]),([
(1,8,13)(2,9,14)(3,10,15)(4,6,11)(5,7,12)]),([[(1,8,14)(2,9,15)(3,10,11)(4,6,12)(5,7,13)]),
([ (1,8,15)(2,9,11)(3,10,12)(4,6,13)(5,7,14)]),([[(1,8,11)(2,9,12)(3,10,13)(4,6,14)(5,7,15)]),
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